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Implications of String Constraints for Exotic Matter and Z' s Beyond the Standard Model

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ABSTRACT: Global consistency of string compactifications places constraints on the chiral matter spectrum of a gauge theory which include those necessary for the absence of cubic nonabelian anomalies, but also contain some additional conditions. In the class of theories we study, some of these are present in a field theory augmented by anomalous $U(1)$'s and Chern-Simons terms, but some are genuinely not present in field theory. Their violation has phenomenological implications, rendering inconsistent many quiver gauge theories with the chiral matter spectrum of the MSSM. The inconsistent MSSM quivers often violate the constraints in a particular way that is suggestive of what matter must be added for consistency. The preferred matter additions are MSSM singlets with anomalous $U(1)$ charge, hyperchargeless $SU(2)$ triplets, quasichiral Higgs or lepton isodoublet pairs, quasichiral quark isosinglet pairs, and nonabelian singlets with charge ± 1 . Smaller numbers of quark isodoublet pairs, lepton pairs with charges $(\pm 1, \pm 2)$, and chiral fourth families are also found. We present the results of systematic analyses including multiplicity counts of matter beyond the standard model and also study the possibility of using the singlets for a dynamical perturbative μ -term or for neutrino mass. We also systematically study the appearance of additional non-anomalous $U(1)'$ symmetries in the low energy theory and find that family non-universality is very common. These new physics effects may be observable at the LHC even for a large string scale M_s close to the Planck scale.

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1 Introduction

String theory is a consistent UV-complete theory that naturally gives rise to gauge theories. This means, in particular, that gauge theories arising from some string theories automatically satisfy field theoretic anomaly cancellation conditions. For example, in type IIA orientifold compactifications¹, gauge theories live on D6-branes which wrap three-cycles in the internal dimensions, while chiral matter appears at the intersection of two D6-branes. Consistency of the compactification imposes constraints on the chiral matter (known as tadpole cancellation conditions) which ensure the cancellation of non-abelian gauge anomalies (i.e., anomalies involving the cube of a single simple group factor). The absence of abelian, mixed, and mixed gauge-gravitational anomalies is also ensured, sometimes utilizing the generalized Green-Schwarz mechanism, in which the triangle anomalies are cancelled by Chern-Simons couplings.

However, global consistency conditions often impose extra constraints which are not present in standard field theoretic gauge theories². This occurs even in the “local” or “bottom-up” approach [10, 11], which allows for the study of issues related to particle

¹For original work on intersecting branes, see [1–6]. For extensive reviews, see [7, 8]. For a shorter introduction to intersecting branes and their non-perturbative D-instanton effects, see [9].

²There is also the well-known distinction that perturbative intersecting-brane constructions only lead to matter fields transforming as bifundamentals, symmetric or antisymmetric products, and adjoints. Gauge singlets can arise as bifundamentals of $U(1)$ or antisymmetrics of $SU(2)$, but in both cases they have anomalous $U(1)$ charge that frequently plays an important phenomenological role.

physics while ignoring global aspects of the compactification, such as the specification of a particular compactification manifold. Despite the fact that it ignores global aspects of a compactification, the bottom-up approach still contains string theoretic input and constraints. Many of the new constraints are captured in “augmented” gauge field theory, i.e., field theory augmented by “anomalous” $U(1)$ symmetries and Chern-Simons couplings³. It is precisely this type of field theory that occurs in many string compactifications.

In this paper we use the language of quiver gauge theories to specify the gauge symmetry and matter content of such a field theory. A quiver is a graph of nodes and directed edges, where the nodes represent gauge factors and directed edges represent chiral matter⁴. In the type II intersecting brane context, the quiver nodes are the gauge symmetries on D-branes, the edges are matter fields living at the intersection of branes, and the anomalous $U(1)$ ’s are the (trace) $U(1)$ of a $U(N)$ gauge symmetry on a brane. The Chern-Simons couplings are a generic feature of the D-brane effective action, and so augmented field theories of the type described are strongly motivated by string theory.

Though many aspects of stringy quiver gauge theories can be captured by considering an augmented field theory, there are additional “stringy” consistency constraints that are not required in field theory (given our current knowledge). The constraints necessary for tadpole cancellation, which ensures the net cancellation of D-brane charge on the compact internal dimensions via Gauss’ law, provide two such examples. The simplest of these requires that (in the absence of symmetric and antisymmetric products) there are an equal number of $\mathbf{2}$ and $\overline{\mathbf{2}}$ representations, where the bar denotes the charge under the trace $U(1)$ of $U(2)$. Though the analogous constraint for $\mathbf{3}$ and $\overline{\mathbf{3}}$ ensures the absence of $SU(3)$ ³ triangle anomalies, the constraint on doublets is not present in field theory, as there is never an $SU(2)$ ³ anomaly. To make this point more concrete, we will present a quiver gauge theory with the exact MSSM spectrum and additional $U(1)$ ’s, the anomalies of which can be cancelled by the introduction of Chern-Simons terms. This quiver does not suffer from any gauge anomalies and appears to have no field theoretic pathology, but does in fact violate the constraint that requires the same number of $\mathbf{2}$ and $\overline{\mathbf{2}}$. The violation is due to the precise structure of the MSSM spectrum with respect to anomalous $U(1)$ ’s, so the constraints have implications for the structure of matter both in and beyond the standard model.

We discuss the constraints in a broad context because, though they have been extensively studied in the context of type IIA orientifold compactifications with intersecting branes, they also appear in other patches of the landscape. This includes type IIB orientifold compactifications, as one might expect via T-duality, but in addition the same constraints appear in the non-geometric rational conformal field theory phase of type II [13–15], complete with a “mod 3” feature that is surprising on first sight. Since the stringy constraints are closely related to the presence of anomalous $U(1)$ factors, it would be interesting to see if they can be derived in smooth heterotic compactifications in the case where

³“Anomalous” symmetries are those in which the apparent anomalies are cancelled by the Chern-Simons terms. Henceforth, we usually omit the parentheses.

⁴For an example of a quiver with the exact MSSM spectrum (i.e., without right-handed neutrinos), see [9]. For a discussion of the advantages of this approach to the string landscape, see [12].

the holomorphic vector bundle V has structure group $U(N)$. Such compactifications are S-dual to type I string compactifications with D9-branes and D5-branes, which in turn are mirror symmetric to the type IIA intersecting braneworlds we typically have in mind [16].

Since the stringy constraints have implications for the structure of matter (with respect to anomalous $U(1)$'s), one might choose to explore the phenomenology of those MSSM quivers which satisfy the constraints necessary for string consistency. At the level of the spectrum, such issues were investigated in [17], which included an analysis of possible MSSM hypercharge embeddings⁵. These identify the hypercharge with a linear combination of anomalous $U(1)$'s which itself is anomaly free. The anomalous $U(1)$ gauge bosons generically obtain a large Stueckelberg mass, but the $U(1)$ symmetries survive as global selection rules in the effective action, forbidding certain superpotential terms. In type II string compactifications, those couplings are forbidden in string perturbation theory, though D-instanton effects can generate non-perturbative corrections [18–20] to otherwise forbidden couplings, such as those involving the μ parameter or a large Majorana seesaw mass for right-handed neutrinos. Quiver analyses at the level of couplings including the implications of D-instantons were carried out in a number of works⁶, beginning with [21]. For MSSM-like quivers, semi-realistic Yukawa couplings and mass hierarchies were studied in [22–25]. Issues of proton decay [24, 25], a non-perturbatively generated Dirac neutrino mass operator $LH_u \nu_L^c$ [26], a non-perturbatively generated Weinberg operator $LLH_u H_u$ [27, 28], and singlet-extended standard models [12] have also been investigated in this context. For an analysis of $SU(5)$ GUT quivers at the level of couplings, including some globally consistent models, see [15].

In this work we take a related but different approach, studying what matter beyond the exact MSSM is preferred by the stringy constraints. We do this by systematically studying what matter could be added to otherwise inconsistent MSSM quivers to render them consistent. Phenomenologically, such an analysis is important because, though the MSSM is minimal, it is by no means the only option: sometimes simple extensions solve phenomenological problems, and in any case additional matter or interactions may survive as apparently accidental remnants of the underlying construction. Matter beyond the standard model is one of the major possibilities for new physics to be observed at the LHC, and it is interesting that the constraints we study prefer some possibilities over others. We emphasize that we do not add matter arbitrarily, but instead in the most general way afforded by the stringy constraints. This work can be viewed as a small step in exploring the interesting subset of the string landscape that is consistent with the SM or MSSM but allows additional TeV-scale physics.

For the simplest class of MSSM quivers, which have three nodes, we will explicitly show that the violation of stringy constraints takes a form which strongly suggests what matter should be added to the quiver for the sake of string consistency. We continue with

⁵[17] also allows for exotics whose mass is not protected by the electroweak scale. In addition to other differences from their work, our work allows for the most general exotics afforded by the brane spectrum, rather than specific subsets.

⁶Previous work on type II motivated MSSM-containing quivers often denoted the left-chiral fields u_L^c , d_L^c , e_L^c , and ν_L^c by u_R , d_R , E_R , and N_R , respectively.

a systematic four-node analysis, demonstrating the “preferences” of the stringy constraints for matter beyond the standard model. The most likely matter additions in the analysis are MSSM singlets charged under the anomalous $U(1)$ ’s, triplets of $SU(2)$ with weak hypercharge $Y = 0$, quasichiral Higgs or lepton isodoublet pairs, quasichiral quark isosinglet pairs, and nonabelian singlets with charge ± 1 . (By quasichiral pairs we mean fields that transform as vector pairs under the SM gauge group, but which are chiral under additional anomalous or non-anomalous $U(1)$ factors.) Quasichiral $d_L^c \bar{d}_L^c$ pairs are far more likely than their up-type quark counterparts. Even less common than the latter are quasichiral quark isodoublet pairs and lepton isodoublet pairs with charges $(\pm 1, \pm 2)$. Often the SM singlets couple perturbatively to the quasichiral pairs in such a way as to generate masses for the pair when the singlet acquires a vacuum expectation value (VEV), the most familiar example being a dynamical μ -term for a Higgs pair.

There are also a number of additions that correspond to an ordinary chiral fourth family, or to a chiral family in which the electric charges are shifted by ± 1 . The SM chiral extensions lead to large corrections to precision electroweak observables [29–31] except for small parameter regions with delicate cancellations. More seriously, experimental limits on their masses require large Yukawa couplings to the Higgs doublets, leading to Landau poles at one (two) loops for the Yukawa (gauge) couplings at low (e.g., 10-1000 TeV) scales [32], effectively eliminating the possibility of any perturbative connection to the underlying string construction unless the string scale is very low. We will therefore focus on the quasichiral pairs⁷, which do not affect precision observables unless there are large mass splittings or mixings, and which can acquire large masses by the couplings to MSSM singlet fields⁸, avoiding the problem of Landau poles.

We present results for the likelihood of all matter fields allowed in the analysis and also for other phenomenological considerations, such as distinguishing between various types of singlets.

Many extensions of the SM involve additional $U(1)'$ gauge symmetries in the low energy theory. These include not only string constructions, but also many grand unified theories, alternative theories of electroweak breaking, etc. The associated Z' gauge bosons are excellent candidates for new physics to be observed at the LHC, and the extra gauge symmetry often has implications for extended Higgs, neutralino, and fermion sectors; the μ problem; supersymmetry breaking and mediation; for neutrino mass; and for cosmology. (For recent reviews, see [34, 39].) In the theories we study in this paper, Chern-Simons coupling typically give a string scale Stueckelberg mass to $U(1)$ gauge bosons, though in some cases a non-anomalous $U(1)$ is left massless. We require that one such⁹ non-anomalous $U(1)$ is present and identifiable as weak hypercharge. In some cases there exists another massless $U(1)$ factor, which should be interpreted as a $U(1)'$ gauge symmetry, with a gauge boson that can obtain a low scale mass via the ordinary Higgs mechanism. We

⁷We also exclude from consideration the subset of the four-node quivers which lead to fractionally-charged color-singlet particles.

⁸For LHC implications of such quasichiral pairs, see, e.g., [33–38].

⁹Massless non-anomalous $U(1)$ ’s may also emerge as subgroups of an underlying non-abelian symmetry. In some, but not all, cases the weak hypercharge and/or additional $U(1)$ ’s considered here may emerge in this way, e.g., by the splitting between the $SU(5)$ branes in a $U(5)$ stack of D6-branes.

systematically study the appearance of $U(1)'$ factors in three-node and four-node quivers, which are typically accompanied by quasichiral exotics to ensure anomaly cancellation. In many cases the $U(1)'$ couplings are not family universal. This implies flavor changing Z' couplings, such as those which have been suggested as one explanation of possible anomalies in the neutral B system [39–44].

We emphasize that the additional Z' s that we are considering do not acquire Stuckelberg masses and are therefore massless except for any mass obtained by the ordinary Higgs mechanism. In this case, there can be a light Z' even if the string scale in a given compactification is $\mathcal{O}(M_{pl})$. This is in contrast to a different scenario [45–54] in which the light Z' is associated with an anomalous $U(1)$ which obtains a Stuckelberg mass at the string scale, which is assumed to be $\mathcal{O}(\text{TeV})$. While extremely interesting, there is not a strong reason to expect such a low string scale.

This paper is organized as follows. In section 2 we elaborate on the stringy constraints, and give an example of a quiver which is consistent field theoretically, but does not satisfy the string tadpole conditions. In section 3, we study all three-node quivers with the exact MSSM spectrum, systematically adding up to five additional fields to those of the MSSM in order to satisfy the stringy conditions. We explicitly exclude vector pairs of fields (i.e., vector under both the gauge symmetries and anomalous $U(1)$'s), because such pairs always satisfy the stringy constraints and typically acquire¹⁰ string scale masses. These three-node quivers are not realistic, because with our assumptions the lepton and down-type Higgs doublets are indistinguishable, implying significant violation of lepton number and R-parity. Nevertheless, they illustrate other interesting issues, such as the types of additional matter beyond the MSSM required by the stringy conditions, and the possibility of an additional Z' with family non-universal couplings. In section 4 we systematically study the consistent four-node quivers for each realistic hypercharge embedding, again allowing up to 5 matter additions but no vector pairs. Most of the matter additions are MSSM singlets (with anomalous $U(1)$ charges), isotriplets with $Y = 0$, or quasichiral pairs, which are non-chiral under the MSSM gauge group. The largest number of the latter are lepton/Higgs doublets, down-type isosinglet quarks, and nonabelian singlets with charge ± 1 . There are smaller numbers of pairs of up-type quark isosinglets, quark isodoublets, and lepton/Higgs doublets with charges $(\pm 1, \pm 2)$. There are also examples of chiral ordinary or charge-shifted fourth families, and (as is typical in string theory) of fractionally charged color singlets (or particles which can form fractionally charged color-singlet bound states). We further examine the numbers of quivers with additional properties, including a distinction between lepton and down-type Higgs doublets at the quiver level (necessary but not sufficient for lepton number conservation), how many MSSM-singlets S_μ can have perturbative couplings $S_\mu H_u H_d$ that can lead to an NMSSM-type dynamical μ term, and how many can have the types of Dirac couplings needed for right-handed neutrinos in a seesaw model. We also examine the subset of quivers involving a non-anomalous $U(1)'$ gauge symmetry (in addition to hypercharge), with a Z' that does not acquire a Stuckelberg mass (i.e., which might be observable at the

¹⁰Vector pairs have large masses at a generic point in moduli space, though at special points they may become massless.

LHC even for a very large string scale comparable to the Planck scale), and find that the majority of these involve family non-universal couplings for at least one kind of fermion (because the families are quiver distinct), leading to FCNC effects that might be relevant to the neutral B system. In appendix A we sketch the origin and structure of the stringy tadpole and $U(1)$ masslessness conditions, while appendix B contains tables of the possible fields that can occur for each type of quiver.

2 Stringy Constraints and Augmented Field Theory

The constraints on chiral matter necessary for consistency of a type II orientifold (or related) string compactification are discussed in detail in appendix A, along with a brief sketch of their derivation. The constraints include:

- **Field Theory Anomaly Constraints:** These include the cancellation of non-abelian triangle anomalies, as well as the abelian, mixed, and mixed gauge-gravitational anomalies for non-anomalous $U(1)$'s, such as the hypercharge.
- **Augmented Field Theory Constraints:** These are constraints arising in some more specialized (augmented) field theory. One example would be the constraints that are required for the cancellation of triangle and gravitational anomalies by Chern-Simons terms in the MSSM augmented by anomalous $U(1)$'s. In practice, augmented field theory constraints can contain stringy inputs or motivations, such as the consideration of the anomalous $U(1)$'s and the choices of representations considered.
- **Stringy Constraints:** These are constraints necessary for string consistency that are not required in field theory, given our current knowledge.

The gauge theories common in type II string compactifications can contain non-anomalous nonabelian and abelian factors, as in the standard model, but also generically include anomalous $U(1)$ factors corresponding to the trace generator of a $U(N)$ factor. Fields of the same standard model representation can be charged differently with respect to the anomalous $U(1)$'s, corresponding to the appearance at the intersection of different pairs of D-branes. We refer to such fields as “quiver-distinct”. In addition, the Wess-Zumino component of the D-brane worldvolume action provides Chern-Simons couplings which, after dimensional reduction, cancel the four-dimensional abelian and mixed anomalies associated with the anomalous $U(1)$'s.

It is possible, of course, to consider a gauge theory with anomalous $U(1)$ factors without any reference to string theory. The anomalies can be cancelled by the introduction of four-dimensional Chern-Simons terms of the form $\int \phi F \wedge F$, $\int \phi R \wedge R$ and $\int B \wedge F$, where F is the field strength of an anomalous $U(1)$ gauge boson and the 0-form ϕ and 2-form B both possess an axionic shift symmetry. This approach was studied in [55, 56], where [55] gave the generic solution for the structure of Chern-Simons coefficients that cancel all abelian and mixed gauge anomalies. That study found no qualitative difference between string theory and augmented field theory. It is also well known that terms of the form $\int \phi R \wedge R$

can cancel mixed abelian-gravitational anomalies [5]. The constraints on Chern-Simons coefficients that ensure the cancellation of anomalies are therefore augmented field theory constraints¹¹.

There are two sets of constraints which we study in this paper, which are necessary for string consistency and a massless hypercharge boson. We state them here and discuss them in the context of field theory, augmented field theory, and stringy constraints, but refer the reader to appendix A for a more in depth discussion of their string theoretic origin. The well-known tadpole cancellation conditions of type II string theory impose a constraint on the cycles wrapped by D-branes which ensures the net cancellation of D-brane charge in the compact internal space via Gauss' law. The condition on D-brane cycles imposes constraints on chiral matter, given by

$$\begin{aligned} N_a \geq 2 : \quad & \#a - \#\bar{a} + (N_a + 4) (\#\square\square_a - \#\overline{\square\square}_a) + (N_a - 4) (\#\square_a - \#\overline{\square}_a) = 0 \\ N_a = 1 : \quad & \#a - \#\bar{a} + (N_a + 4) (\#\square\square_a - \#\overline{\square\square}_a) = 0 \pmod{3}, \end{aligned} \quad (2.1)$$

where N_a refers to a stack of N_a coincident branes with $U(N_a)$ gauge symmetry, a and \bar{a} are fundamentals and antifundamentals of $U(N_a)$, and Young Tableaux denotes symmetric and antisymmetric representations as usual. For $N_a > 2$ these are simply the conditions which ensure the absence of $SU(N_a)^3$ triangle anomalies, but the constraints for $N_a = 2$ and $N_a = 1$ are stringy constraints.

The other constraints we study are related to Chern-Simons coupling of the form $\int B \wedge F$, which give a Stuckelberg mass to anomalous $U(1)$ gauge bosons (and sometimes even non-anomalous bosons). If these couplings are absent for some non-anomalous linear combination $\sum q_x U(1)_x$, the corresponding boson does not obtain a Stuckelberg mass. The conditions on the chiral matter

$$-q_a N_a (\#\square\square_a - \#\overline{\square\square}_a + \#\square_a - \#\overline{\square}_a) + \sum_{x \neq a} q_x N_x (\#(a, \bar{x}) - \#(a, x)) = 0 \quad (2.2)$$

for $N_a \geq 2$, and

$$-q_a \frac{\#a - \#\bar{a} + 8(\#\square\square_a - \#\overline{\square\square}_a)}{3} + \sum_{x \neq a} q_x N_x (\#(a, \bar{x}) - \#(a, x)) = 0 \quad (2.3)$$

for $N_a = 1$ and the constraints (2.1) are necessary but not sufficient for a massless gauge boson and for tadpole cancellation, respectively. We require that the masslessness constraints are satisfied by a linear combination identifiable as weak hypercharge. These constraints and the constraints (2.1) are necessary but not sufficient for a massless hypercharge boson and tadpole cancellation, respectively.

¹¹Over the last few years, similar studies of augmented field theories and their relation to string theories (including constraints) were begun in [57–59], which focus on the relationship between $6D$ supergravity theories and $6D$ F-theory compactifications. These theories do not contain the $4D$ chiral matter which we are interested in.

Let us consider a particular quiver gauge theory with respect to these constraints. It is a three-node quiver, which means that it has three gauge factors, here taken to be $U(3)_a \times U(2)_b \times U(1)_c$. For simplicity, we take all of the three families to have the structure

$$\begin{aligned} q_L : (a, \bar{b})_{1,-1,0} \quad u_L^c : (\bar{a}, \bar{c})_{-1,0,-1} \quad d_L^c : (\bar{a}, c)_{-1,0,1} \\ L : (\bar{b}, \bar{c})_{0,-1,-1} \quad e_L^c : (\square\square_c)_{0,0,2}, \end{aligned} \quad (2.4)$$

and take $H_u : (b, c)$ and $H_d : (\bar{b}, \bar{c})$ for the two Higgs doublets. We explicitly denote the charges of the fields under $U(1)_a$, $U(1)_b$ and $U(1)_c$ as subscripts.

In this quiver, a generic abelian symmetry $U(1)_G$ can be written as $U(1)_G = q_a U(1)_a + q_b U(1)_b + q_c U(1)_c$. Though the theory is free of non-abelian anomalies, we wish to study the abelian and mixed anomalies. Cancellation of these anomalies requires

$$\begin{aligned} U(1)_G^3 &\sim 6(q_a - q_b)^3 + 3(-q_a - q_c)^3 + 3(-q_a + q_c)^3 + 2(-q_b - q_c)^3 + (2q_c)^3 \\ &= q_b(-18q_a^2 + 18q_a q_b - 8q_b^2 - 6q_b q_c - 6q_c^2) + q_c^2(-18q_a + 6q_c) = 0 \\ U(1)_G SU(2)^2 &\sim 3(q_a - q_b) + (-q_b - q_c) = 3q_a - q_c - 4q_b = 0 \\ U(1)_G SU(3)^2 &\sim 2(q_a - q_b) + (-q_a - q_c) + (-q_a + q_c) = -2q_b = 0 \\ U(1) \text{ grav} &\sim 6(q_a - q_b) + 3(-q_a - q_c) + 3(-q_a + q_c) + 2(-q_b - q_c) + 2q_c \\ &= -8q_b = 0, \end{aligned} \quad (2.5)$$

for which the only solution has $q_b = 0$ and $3q_a - q_c = 0$. This non-anomalous $U(1)$ can be identified with hypercharge, and is (up to scaling) the Madrid embedding which will be discussed in section 3.2. The theory also has two linearly independent anomalous $U(1)$ symmetries.

By the appropriate introduction of Chern-Simons terms, either in augmented field theory or automatically in a string compactification, all of the anomalies associated with the anomalous $U(1)$'s can be cancelled. After doing so, the theory is completely free of non-abelian, abelian, mixed abelian-gauge and mixed abelian-gravitational anomalies. The spectrum is that of the exact MSSM, so there is no global $SU(2)$ anomaly [60] (i.e. there are not an odd number of $SU(2)$ doublets). From the point of view of augmented field theory, this theory suffers no pathology.

However, this quiver does not satisfy all of the stringy constraints. From (2.4), all standard model fermions (save the vector-like Higgs pair) which are doublets of $SU(2)$ come as \bar{b} rather than b , so that

$$\#b - \#\bar{b} + (6)(\#\square\square_b - \square\square_b) + (-2)(\#\square_b - \bar{\square}_b) = -12. \quad (2.6)$$

The constraint (2.1) with $N_a = 2$, which is necessary for tadpole cancellation, requires that this quantity be zero. Therefore, this quiver does not satisfy the stringy constraints on chiral matter. Since $N_b = 2$ in this quiver, we refer to the violation of this constraint as having net “ T_b -charge”. In analogy, the left hand side of (2.2) and (2.3) are referred as “M-charge” for convenience. Via other three-node quivers, one can use similar arguments

to argue that the $N_a = 1$ constraint of (2.1) is also stringy. Indeed, we will show in section 3 that these constraints are precisely the ones violated by any inconsistent three-node MSSM quiver. We will also show that the structure of the violation strongly suggests what type of matter should be added to the quiver to render it consistent. One would have no need for such matter additions in augmented field theory.

Let us discuss two more examples which shed light on the constraints and their phenomenological importance. The four-node Madrid hypercharge embedding has $U(1)_Y = \frac{1}{6}U(1)_a + \frac{1}{2}U(1)_c + \frac{1}{2}U(1)_d$ and will be systematically studied in section 4. Consider any quiver with the exact MSSM spectrum and the four-node Madrid embedding which satisfies the constraints (2.1), (2.2) and (2.3). Now consider the addition of three fields that transform as (c, \bar{d}) . These are MSSM singlets, and therefore the quiver is still free of $SU(3)^3$, $SU(3)^2 - Y$, $SU(2)^2 - Y$, Y^3 and $Y - G - G$ anomalies. The singlets do not contribute to any mixed anomalies involving hypercharge and anomalous $U(1)$'s, and any mixed abelian anomalies (which necessarily do not involve hypercharge) to which they contribute can be cancelled by Chern-Simons terms. The singlets are perfectly well behaved with respect to field theoretic (anomaly) constraints. However, after the addition of the singlets, the quiver still satisfies the constraints (2.1) and (2.2), but now violates the constraints (2.3), since the M-charge is now $M_c = 1$ and $M_d = -1$. This is striking: these MSSM singlets, which are field theoretically mundane, necessarily force the hypercharge boson to have a Stuckelberg mass. Similarly, the standard model vector pair $(\bar{\mathbf{3}}, 1)_{-\frac{2}{3}}$, $(\mathbf{3}, 1)_{\frac{2}{3}}$ realized as (\bar{a}, \bar{c}) , (a, d) forces the hypercharge boson to obtain a Stuckelberg mass. This is necessary because the theory has $Y U(1)_c U(1)_c$ and $Y U(1)_d U(1)_d$ triangle anomalies, which for anomaly cancellation requires the presence of the term $\int B \wedge F_Y$ which gives the hypercharge boson a mass.

3 Lessons from Three-Node Quivers

An MSSM-containing quiver must have at least three nodes, which we take to have $U(3)_a \times U(2)_b \times U(1)_c$ gauge symmetry, as motivated by type II compactifications. As mentioned in section 2, an anomalous $U(1)$ gauge boson always receives a string scale Stuckelberg mass due to the presence of Chern-Simons couplings which participate in anomaly cancellation, though some times a non-anomalous $U(1)$ will be left massless. If there is precisely one such non-anomalous¹² linear combination

$$U(1)_Y = q_a U(1)_a + q_b U(1)_b + q_c U(1)_c, \quad (3.1)$$

that can be identified as hypercharge, then the low energy gauge symmetry is $SU(3) \times SU(2) \times U(1)_Y$. Ensuring that a $U(1)$ is massless requires that the chiral matter in the theory satisfies the constraints (2.2) and (2.3). For three-node quivers there are only two such linear combinations (known as hypercharge embeddings [17]) which allow for the spectrum of the MSSM. If a three-node quiver has the exact MSSM spectrum, the

¹²In this context, a massless $U(1)$ is always non-anomalous, though a $U(1)$ gauge boson with a Stuckelberg mass could be either anomalous or anomaly free [61].

conditions for a massless hypercharge are always satisfied, and the violation of the tadpole conditions suggests the addition of particular matter fields to render the quiver consistent, i.e., to satisfy the stringy constraints.

Let us first give one illustrative example of the power of stringy constraints which applies to both embeddings. Consider a three-node quiver with the spectrum of the exact MSSM. If the quiver satisfies the stringy constraints, then it is straightforward to show that it must have least one particle $P = (\mathbf{1}, \mathbf{2})_{-\frac{1}{2}}$ which forms a vector pair with H_u under all symmetries of the theory and is therefore present in the superpotential. In string compactifications, such a term will generically have a string scale mass, giving rise to either a very large μ -term $H_d H_u$ or a large lepton violating coupling $L H_u$ depending on whether P is identified as a down-type Higgs or a lepton doublet. Both of these possibilities have significant phenomenological drawbacks, but are the only possibilities allowed by the stringy constraints for a three-node quiver with the spectrum of the exact MSSM. Thus, one requires either additional matter, or some mechanism to suppress these couplings.

3.1 The Madrid Embedding

The first hypercharge embedding we examine is the three-node Madrid embedding [61],

$$U(1)_Y = \frac{1}{6} U(1)_a + \frac{1}{2} U(1)_c. \quad (3.2)$$

It gives rise to the largest number of three-node MSSM quivers, including the quiver presented in section 2; its natural extension by adding a $U(1)_d$ node with $q_d = \frac{1}{2}$ gives rise to the largest number of four-node quivers. A quiver with this hypercharge embedding and appropriate spectrum might arise dynamically from brane splitting of a $U(5) \times U(1)_c$ GUT quiver, with $U(1)_a$ contained in $U(5)$. This would be a flipped $SU(5)$ quiver (e.g., [62]), rather than a Georgi-Glashow $SU(5)$ one [63]. The $\mathbf{10}$, realized as \square_5 , would give rise to d_L^c , rather than u_L^c , upon brane splitting.

All possible ways of realizing each MSSM field are listed in Table 1. Given this information, it is a straightforward exercise to enumerate all three-node MSSM quivers with the Madrid hypercharge embedding. Every such quiver satisfies the conditions necessary for a massless hypercharge. The constraints (2.1) necessary for tadpole cancellation are not always satisfied, however. The a -node, b -node, and c -node have $N_a = 3, 2, 1$ in (2.1), respectively, and we label the value of the left-hand side T_a , T_b , and T_c . A simple computation shows that any three-node MSSM quiver in the Madrid embedding has T_a , T_b , and T_c of the form

$$T_a = 0 \quad T_b = \pm 2n \quad T_c = 0 \bmod 3 \quad \text{with } n \in \{0, \dots, 7\}. \quad (3.3)$$

However, a consistent quiver must have $(T_a, T_b, T_c) = (0, 0, 0 \bmod 3)$ to satisfy (2.1). Thus, though T_a and T_c satisfy the constraint, T_b only does so for quivers with $n = 0$. We henceforth refer to such overshooting as having net “T-charge”, and the analogous overshooting in the conditions (2.2) and (2.3) ensuring a massless hypercharge as having net “M-charge”.

The T-charges and M-charge of each possible realization of MSSM fields are given in Table 1.

Field	Transformation	T_a	T_b	T_c	M_a	M_b	M_c
q_L	(a, \bar{b})	2	-3	0	0	$-\frac{1}{2}$	0
q_L	(a, b)	2	3	0	0	$-\frac{1}{2}$	0
u_L^c	(\bar{a}, \bar{c})	-1	0	-3	$\frac{1}{2}$	0	1
d_L^c	\square_a	-1	0	0	$-\frac{1}{2}$	0	0
d_L^c	(\bar{a}, c)	-1	0	3	$-\frac{1}{2}$	0	0
H_u	(b, c)	0	1	2	0	$-\frac{1}{2}$	$-\frac{1}{3}$
H_u	(\bar{b}, c)	0	-1	2	0	$-\frac{1}{2}$	$-\frac{1}{3}$
L	(b, \bar{c})	0	1	-2	0	$\frac{1}{2}$	$\frac{1}{3}$
L	(\bar{b}, \bar{c})	0	-1	-2	0	$\frac{1}{2}$	$\frac{1}{3}$
e_L^c	\square_c	0	0	5	0	0	$-\frac{4}{3}$

Table 1. All possible MSSM field transformations for the hypercharge embedding $U(1)_Y = \frac{1}{6}U(1)_a + \frac{1}{2}U(1)_c$, along with their contributions to the conditions necessary for tadpole cancellation and a massless $U(1)_Y$. Here and in Table 4 the rows labeled L represent possible assignments for either the lepton doublets or for the H_d , which have the same MSSM quantum numbers. The two assignments for H_u lead to equivalent quivers, so we will choose (b, c) .

The form of tadpole overshooting in (3.3) strongly suggests what type of matter should be added to the (inconsistent) quivers with $n \neq 0$. Interestingly, the fact that T_b in (3.7) is even ensures that any matter added to a three-node MSSM Madrid quiver to render it consistent will not introduce a global $SU(2)$ anomaly¹³. The possible additional fields that can be added to the quiver are listed in Table 11 in appendix B. One sees that the only allowed single-particle additions are MSSM singlets with anomalous $U(1)$ charge or $SU(2)$ triplets with $Y = 0$, since any of the other fields with $T_b \neq 0$ would induce T-charge on another node. Two-particle additions could include some combination of these fields, but might also include quasichiral Higgs, lepton, or quark doublet pairs.

We have performed a systematic analysis of all allowed sets of matter additions with up to five fields and present the results for the Madrid embedding in Table 2. An “allowed” matter addition to an existing quiver must cancel any T-charge or M-charge. We also require that a quiver with matter additions be completely absent of vector pairs¹⁴, by which we mean vector with respect to all symmetries in the theory, including anomalous $U(1)$ ’s. As mentioned above, this assumption eliminates three-node quivers with no additions. Finally, for a three-node quiver in the Madrid embedding there is an equivalent quiver¹⁵

¹³See appendix A for some brief, but general, comments about global $SU(2)$ anomalies in this context.

¹⁴This is because vector pairs are allowed in string perturbation theory, but obtain string scale masses at generic points in moduli space, in which case they can be integrated out, i.e., they “vector up”. Often times MSSM quivers Q_1 and Q_2 with matter additions A_1 and A_2 will be equivalent due to change in family structure from vectoring up. A vector pair can also be added arbitrarily to a quiver, since they have no T-charge or M-charge, and thus aren’t particularly important for the purposes of this work.

¹⁵In a type II string compactification, this happens by switching which brane in an orientifold invariant pair is designated to be the image brane.

obtained by replacing $b \leftrightarrow \bar{b}$. We remove this overcounting by fixing H_u to transform as (b, c) . Similar statements apply to the other embeddings.

A major difficulty with the three-node quivers is that the lepton doublets L and down-type Higgs doublet H_d are indistinguishable in both their gauge and anomalous $U(1)$ quantum numbers, because once we exclude vector pairs they have only one allowed quiver assignment. This implies that lepton number (and R-parity) violating couplings such as QLd_L^c , LLe_L^c , and LH_u are allowed at the same level (perturbatively or by non-perturbative D-instanton effects) as the corresponding lepton-number conserving couplings $QH_d d_L^c$, $LH_d e_L^c$, and $H_d H_u$. We ignore this difficulty in this section, as we are more concerned with the gauge quantum numbers of the matter additions. However, we will discuss it in more detail in section 4, where it will be seen that many of the four-node quivers do allow the desired distinction between the lepton and Higgs fields.

We see from the results of the table that a large number of quivers indeed involve MSSM singlets and triplets¹⁶ of $SU(2)$ with $Y = 0$. As emphasized previously, these are always charged under the anomalous $U(1)$'s, which strongly affects the structure of couplings. Some of the singlets are candidates for NMSSM-type Higgs singlets or for right-handed neutrinos, depending on the L and H_d assignments, as will be discussed in section 4. Quasichiral quark doublet pairs and down-type quark isosinglet pairs are quite common, but there are no up-type isosinglet pairs. Quasichiral $SU(2)$ -doublet pairs, which may be interpreted as additional Higgs pairs, lepton pairs, or both, are also common, though perhaps not as much as one might expect. This is because in the three-node Madrid embedding the only way to add quasichiral Higgs pairs without any vectoring up is to add more copies of the MSSM H_u and H_d fields. Two of the quivers involve additions which have the MSSM quantum numbers of a $\mathbf{5}+\mathbf{5}^*+\mathbf{1}$ of $SU(5)$. They therefore would not modify the MSSM-type running of the gauge coupling constants at one loop¹⁷. There are also five additions which correspond to a fourth generation $q_L, u_L^c, d_L^c, L, e_L^c$. There are no unusually charged states that could lead to doubly or fractionally charged color singlets.

Of the 105 MSSM quivers whose matter additions are presented in Table 2, only one allows for an additional (non-anomalous) massless $U(1)'$ gauge symmetry. This quiver is presented in Table 3. In this quiver, any linear combination of the form

$$U(1)_{\text{massless}} = aU(1)_a + bU(1)_b + (3a - 2b)U(1)_c \quad (3.4)$$

is left massless¹⁸ by the Green-Schwarz mechanism, where hypercharge is the linear combination with $a = \frac{1}{6}$ and $b = 0$. This quiver exhibits a number of features that we will see are characteristic of $U(1)'$ symmetries in many quivers with a higher number of nodes, such as family non-universality of the $U(1)'$ and an interesting structure of exotics. It has

¹⁶There is no quiver with only an added triplet, since this would require the presence of a vector pair $H_u L$ or $H_u H_d$.

¹⁷However, as is well known, intersecting brane constructions generically do not lead to simple gauge unification conditions at the string or GUT scale [7–9].

¹⁸Technically, this $U(1)$ only satisfies constraints necessary (but not sufficient) for a massless gauge boson. In a globally defined compactification, the $U(1)$ may still become massive, though this possibility cannot be addressed at the quiver level. This is true of all “massless” $U(1)$'s in this paper.

Multiplicity	Matter Additions				
4	$\square\square_b, (1, 3)_0$	$\square\square_b, (1, 3)_0$	$\square_b, (1, 1)_0$	$(a, \bar{b}), (3, 2)_{\frac{1}{6}}$	$(\bar{a}, \bar{b}), (\bar{3}, 2)_{-\frac{1}{6}}$
4	$\square\square_b, (1, 3)_0$	$\square_b, (1, 1)_0$			
4	$\square\square_b, (1, 3)_0$	$\square_b, (1, 1)_0$			
4	$\square\square_b, (1, 3)_0$	$\square_b, (1, 1)_0$	$\square_b, (1, 1)_0$	$(b, \bar{c}), (1, 2)_{-\frac{1}{2}}$	$(b, c), (1, 2)_{\frac{1}{2}}$
4	$\square\square_b, (1, 3)_0$	$\square_b, (1, 1)_0$	$\square_b, (1, 1)_0$	$(b, \bar{c}), (1, 2)_{-\frac{1}{2}}$	$(b, c), (1, 2)_{\frac{1}{2}}$
4	$\square\square_b, (1, 3)_0$	$\square_b, (1, 1)_0$	$\square_b, (1, 1)_0$	$(a, \bar{b}), (3, 2)_{\frac{1}{6}}$	$(\bar{a}, \bar{b}), (\bar{3}, 2)_{-\frac{1}{6}}$
4	$\square_b, (1, 1)_0$	$\square_b, (1, 1)_0$			
4	$\square_b, (1, 1)_0$	$(b, \bar{c}), (1, 2)_{-\frac{1}{2}}$	$(b, c), (1, 2)_{\frac{1}{2}}$		
4	$(b, \bar{c}), (1, 2)_{-\frac{1}{2}}$	$(b, \bar{c}), (1, 2)_{-\frac{1}{2}}$	$(b, c), (1, 2)_{\frac{1}{2}}$	$(b, c), (1, 2)_{\frac{1}{2}}$	
4	$(a, \bar{b}), (3, 2)_{\frac{1}{6}}$	$\square_a, (\bar{3}, 1)_{\frac{1}{3}}$	$(b, \bar{c}), (1, 2)_{-\frac{1}{2}}$	$(\bar{a}, \bar{c}), (\bar{3}, 1)_{-\frac{2}{3}}$	$\square\square_c, (1, 1)_1$
4	$\square\square_b, (1, 3)_0$	$\square_b, (1, 1)_0$	$\square_b, (1, 1)_0$	$\square_b, (1, 1)_0$	$\square_b, (1, 1)_0$
4	$\square\square_b, (1, 3)_0$	$\square_b, (1, 1)_0$	$\square_b, (1, 1)_0$	$\square_b, (1, 1)_0$	$\square_b, (1, 1)_0$
4	$\square\square_b, (1, 3)_0$	$\square_b, (1, 1)_0$	$\square_b, (1, 1)_0$		
4	$\square\square_b, (1, 3)_0$	$\square_b, (1, 1)_0$	$(b, \bar{c}), (1, 2)_{-\frac{1}{2}}$	$(b, c), (1, 2)_{\frac{1}{2}}$	
4	$\square\square_b, (1, 3)_0$	$(b, \bar{c}), (1, 2)_{-\frac{1}{2}}$	$(b, \bar{c}), (1, 2)_{-\frac{1}{2}}$	$(b, c), (1, 2)_{\frac{1}{2}}$	$(b, c), (1, 2)_{\frac{1}{2}}$
4	$\square_b, (1, 1)_0$				
4	$\square_b, (1, 1)_0$	$\square_b, (1, 1)_0$	$(b, \bar{c}), (1, 2)_{-\frac{1}{2}}$	$(b, c), (1, 2)_{\frac{1}{2}}$	
4	$\square\square_b, (1, 3)_0$	$\square\square_b, (1, 3)_0$	$\square_b, (1, 1)_0$	$\square_b, (1, 1)_0$	
4	$\square\square_b, (1, 3)_0$	$\square\square_b, (1, 3)_0$	$\square_b, (1, 1)_0$	$(b, \bar{c}), (1, 2)_{-\frac{1}{2}}$	$(b, c), (1, 2)_{\frac{1}{2}}$
4	$\square_b, (1, 1)_0$	$\square_b, (1, 1)_0$	$\square_b, (1, 1)_0$	$\square_b, (1, 1)_0$	
4	$\square\square_b, (1, 3)_0$	$\square\square_b, (1, 3)_0$	$\square\square_b, (1, 3)_0$	$\square_b, (1, 1)_0$	$\square_b, (1, 1)_0$
4	$\square\square_b, (1, 3)_0$	$\square\square_b, (1, 3)_0$	$\square_b, (1, 1)_0$		
1	$\square_a, (\bar{3}, 1)_{\frac{1}{3}}$	$\square\square_b, (1, 3)_0$	$\square_b, (1, 1)_0$	$(a, \bar{c}), (3, 1)_{-\frac{1}{3}}$	
1	$\square_a, (3, 1)_{-\frac{1}{3}}$	$\square\square_b, (1, 3)_0$	$\square_b, (1, 1)_0$	$(\bar{a}, c), (\bar{3}, 1)_{\frac{1}{3}}$	
1	$\square_a, (\bar{3}, 1)_{\frac{1}{3}}$	$\square\square_b, (1, 3)_0$	$\square_b, (1, 1)_0$	$(a, \bar{c}), (3, 1)_{-\frac{1}{3}}$	
1	$\square_a, (3, 1)_{-\frac{1}{3}}$	$\square\square_b, (1, 3)_0$	$\square_b, (1, 1)_0$	$(\bar{a}, c), (\bar{3}, 1)_{\frac{1}{3}}$	
1	$\square_a, (\bar{3}, 1)_{\frac{1}{3}}$	$\square_b, (1, 1)_0$	$\square_b, (1, 1)_0$	$(a, \bar{c}), (3, 1)_{-\frac{1}{3}}$	
1	$\square_a, (3, 1)_{-\frac{1}{3}}$	$\square_b, (1, 1)_0$	$\square_b, (1, 1)_0$	$(\bar{a}, c), (\bar{3}, 1)_{\frac{1}{3}}$	
1	$\square_a, (\bar{3}, 1)_{\frac{1}{3}}$	$\square_b, (1, 1)_0$	$(b, \bar{c}), (1, 2)_{-\frac{1}{2}}$	$(b, c), (1, 2)_{\frac{1}{2}}$	$(a, \bar{c}), (3, 1)_{-\frac{1}{3}}$
1	$\square_a, (3, 1)_{-\frac{1}{3}}$	$\square_b, (1, 1)_0$	$(b, \bar{c}), (1, 2)_{-\frac{1}{2}}$	$(b, c), (1, 2)_{\frac{1}{2}}$	$(\bar{a}, c), (\bar{3}, 1)_{\frac{1}{3}}$
1	$(a, \bar{b}), (3, 2)_{\frac{1}{6}}$	$(b, \bar{c}), (1, 2)_{-\frac{1}{2}}$	$(\bar{a}, c), (\bar{3}, 1)_{\frac{1}{3}}$	$(\bar{a}, \bar{c}), (\bar{3}, 1)_{-\frac{2}{3}}$	$\square\square_c, (1, 1)_1$
1	$\square_a, (\bar{3}, 1)_{\frac{1}{3}}$	$\square\square_b, (1, 3)_0$	$\square_b, (1, 1)_0$	$\square_b, (1, 1)_0$	$(a, \bar{c}), (3, 1)_{-\frac{1}{3}}$
1	$\square_a, (3, 1)_{-\frac{1}{3}}$	$\square\square_b, (1, 3)_0$	$\square_b, (1, 1)_0$	$\square_b, (1, 1)_0$	$(\bar{a}, c), (\bar{3}, 1)_{\frac{1}{3}}$
1	$\square_a, (\bar{3}, 1)_{\frac{1}{3}}$	$\square_a, (\bar{3}, 1)_{\frac{1}{3}}$	$\square_b, (1, 1)_0$	$(a, \bar{c}), (3, 1)_{-\frac{1}{3}}$	$(a, \bar{c}), (3, 1)_{-\frac{1}{3}}$
1	$\square_a, (3, 1)_{-\frac{1}{3}}$	$\square_a, (3, 1)_{-\frac{1}{3}}$	$\square_b, (1, 1)_0$	$(\bar{a}, c), (\bar{3}, 1)_{\frac{1}{3}}$	$(\bar{a}, c), (\bar{3}, 1)_{\frac{1}{3}}$
1	$\square_a, (\bar{3}, 1)_{\frac{1}{3}}$	$\square_b, (1, 1)_0$	$(a, \bar{c}), (3, 1)_{-\frac{1}{3}}$		
1	$\square_a, (3, 1)_{-\frac{1}{3}}$	$\square_b, (1, 1)_0$	$(\bar{a}, c), (\bar{3}, 1)_{\frac{1}{3}}$		
1	$\square_a, (\bar{3}, 1)_{\frac{1}{3}}$	$\square\square_b, (1, 3)_0$	$\square\square_b, (1, 3)_0$	$\square_b, (1, 1)_0$	$(a, \bar{c}), (3, 1)_{-\frac{1}{3}}$
1	$\square_a, (3, 1)_{-\frac{1}{3}}$	$\square\square_b, (1, 3)_0$	$\square\square_b, (1, 3)_0$	$\square_b, (1, 1)_0$	$(\bar{a}, c), (\bar{3}, 1)_{\frac{1}{3}}$

Table 2. Matter additions and their multiplicity for the 105 three-node quivers with the Madrid embedding, up to five additions, and no vector pairs. Listed for each set of additions is its multiplicity and the field transformation behavior with respect to both the quiver and the MSSM.

some nice phenomenological properties, such as a perturbatively realized top-quark mass for two generations and superpotential couplings which allow for a perturbative dynamical

μ -term

$$W \supset (S_1 + S_2) H_u H_d, \quad (3.5)$$

where S_1 and S_2 are the two singlets added to the quiver for consistency, realized as antisymmetrics of $SU(2)$. As with the other 3-node quivers, however, the leptons L are indistinguishable from H_d .

q_L	$(a, \bar{b})_{1,-1}$	$(a, \bar{b})_{1,-1}$	$(a, b)_{1,1}$
u_L^c	$(\bar{a}, \bar{c})_{-4,2}$	$(\bar{a}, \bar{c})_{-4,2}$	$(\bar{a}, \bar{c})_{-4,2}$
d_L^c	$(\bar{\square}_a)_{2,0}$	$(\bar{a}, c)_{2,-2}$	$(\bar{a}, c)_{2,-2}$
L	$(b, \bar{c})_{-3,3}$	$(b, \bar{c})_{-3,3}$	$(b, \bar{c})_{-3,3}$
e_L^c	$(\square_c)_{6,-4}$	$(\square_c)_{6,-4}$	$(\square_c)_{6,-4}$
H_u	$(b, c)_{3,-1}$		
H_d	$(b, \bar{c})_{-3,3}$		
Add	$(\square_b)_{0,-2}$	$(\bar{\square}_b)_{0,-2}$	$(\bar{\square}_b)_{0,-2}$

Table 3. The only three-node quiver with a $U(1)'$ symmetry with up to five matter additions that contains the MSSM spectrum and has no vector pairs. “Add” refers to the additional matter that must be added to satisfy the stringy constraints. Subscripts denote the values of a and b in (3.4) for that field.

3.2 The non-Madrid Embedding

The other three-node hypercharge embedding, which we call the “non-Madrid” embedding, is given by

$$U(1)_Y = -\frac{1}{3} U(1)_a - \frac{1}{2} U(1)_b. \quad (3.6)$$

As mentioned in the introduction, this could actually emerge from a non-abelian Georgi-Glashow $SU(5)$ subgroup of $U(5)$ by brane splitting, with the u_L^c realized as \square_a in the **10**. Every possible field which might appear in a three-stack quiver is displayed in Table 11. Of those, the ones which correspond to MSSM fields are listed in Table 4.

Enumerating every possible three-node quiver with the non-Madrid embedding and the MSSM spectrum, the overshoot in the tadpole takes the form

$$T_a = 0 \quad T_b = 0 \quad T_c \in \{-15, -11, -9, -7, -5, -3, -1, 1, 3, 5, 7, 9, 11, 13, 15, 19\}. \quad (3.7)$$

Those quivers with $T_c \neq 0 \bmod 3$ violate the string consistency constraints (2.1). The only possible single-particle addition is an MSSM singlet. The 41 quivers from a systematic analysis with up to five matter additions are presented in Table 5. We fix H_u to transform as (\bar{b}, c) in order to account for the quiver symmetry $c \leftrightarrow \bar{c}$. There are many MSSM-singlets, many quasichiral Higgs/lepton doublet pairs, and many quasichiral down-type quark isosinglet pairs. Two of the quivers involve additional matter with the MSSM quantum numbers of $\mathbf{5} + \mathbf{5}^* + \mathbf{1}$ of $SU(5)$, and there are five examples of a fourth family. None have a massless linear combination of anomalous $U(1)$ ’s that can be interpreted as a massless $U(1)'$ symmetry. Similar to the three-node Madrid embedding, there is no distinction between the quantum numbers of the lepton and down-Higgs doublets.

Field	Transformation	T_a	T_b	T_c	M_a	M_b	M_c
q_L	(a, \bar{b})	2	-3	0	-1	1	0
u_L^c	\square_a	-1	0	0	1	0	0
d_L^c	(\bar{a}, c)	-1	0	3	0	0	-1
d_L^c	(\bar{a}, \bar{c})	-1	0	-3	0	0	-1
H_u	(\bar{b}, c)	0	-1	2	0	0	-1
H_u	(\bar{b}, \bar{c})	0	-1	2	0	0	-1
L	(b, \bar{c})	0	1	-2	0	0	1
L	(b, c)	0	1	2	0	0	1
e_L^c	\square_b	0	2	0	0	-1	0

Table 4. All possible MSSM field transformations for the hypercharge embedding $U(1)_Y = -\frac{1}{3}U(1)_a - \frac{1}{2}U(1)_b$, along with their contributions to the conditions necessary for tadpole cancellation and a massless $U(1)_Y$.

Multiplicity	Matter Additions				
4	$\square_c, (1, 1)_0$				
4	$\square_c, (1, 1)_0$	$\square_c, (1, 1)_0$	$\square_c, (1, 1)_0$	$\square_c, (1, 1)_0$	
4	$\overline{\square}_c, (1, 1)_0$	$\overline{\square}_c, (1, 1)_0$			
4	$\overline{\square}_c, (1, 1)_0$	$\overline{\square}_c, (1, 1)_0$	$\overline{\square}_c, (1, 1)_0$	$\overline{\square}_c, (1, 1)_0$	$\overline{\square}_c, (1, 1)_0$
4	$(a, \bar{b}), (\mathbf{3}, \mathbf{2})_{\frac{1}{6}}$	$\square_a, (\bar{\mathbf{3}}, \mathbf{1})_{-\frac{2}{3}}$	$(b, c), (\mathbf{1}, \mathbf{2})_{-\frac{1}{2}}$	$(\bar{a}, c), (\bar{\mathbf{3}}, \mathbf{1})_{\frac{1}{3}}$	$\square_b, (\mathbf{1}, \mathbf{1})_1$
4	$(\bar{b}, c), (\mathbf{1}, \mathbf{2})_{\frac{1}{2}}$	$(\bar{b}, c), (\mathbf{1}, \mathbf{2})_{\frac{1}{2}}$	$(b, c), (\mathbf{1}, \mathbf{2})_{-\frac{1}{2}}$	$(b, c), (\mathbf{1}, \mathbf{2})_{-\frac{1}{2}}$	
4	$(\bar{b}, c), (\mathbf{1}, \mathbf{2})_{\frac{1}{2}}$	$(b, c), (\mathbf{1}, \mathbf{2})_{-\frac{1}{2}}$	$\square_c, (\mathbf{1}, \mathbf{1})_0$	$\square_c, (\mathbf{1}, \mathbf{1})_0$	
4	$(\bar{b}, c), (\mathbf{1}, \mathbf{2})_{\frac{1}{2}}$	$(b, c), (\mathbf{1}, \mathbf{2})_{-\frac{1}{2}}$	$\overline{\square}_c, (\mathbf{1}, \mathbf{1})_0$		
1	$\square_c, (\mathbf{1}, \mathbf{1})_0$	$(\bar{a}, c), (\bar{\mathbf{3}}, \mathbf{1})_{\frac{1}{3}}$	$(\bar{a}, c), (\bar{\mathbf{3}}, \mathbf{1})_{\frac{1}{3}}$	$(a, c), (\mathbf{3}, \mathbf{1})_{-\frac{1}{3}}$	$(a, c), (\mathbf{3}, \mathbf{1})_{-\frac{1}{3}}$
1	$\square_c, (\mathbf{1}, \mathbf{1})_0$	$(a, \bar{c}), (\mathbf{3}, \mathbf{1})_{-\frac{1}{3}}$	$(a, \bar{c}), (\mathbf{3}, \mathbf{1})_{-\frac{1}{3}}$	$(\bar{a}, \bar{c}), (\bar{\mathbf{3}}, \mathbf{1})_{\frac{1}{3}}$	$(\bar{a}, \bar{c}), (\bar{\mathbf{3}}, \mathbf{1})_{\frac{1}{3}}$
1	$\square_c, (\mathbf{1}, \mathbf{1})_0$	$(\bar{a}, c), (\bar{\mathbf{3}}, \mathbf{1})_{\frac{1}{3}}$	$(a, c), (\mathbf{3}, \mathbf{1})_{-\frac{1}{3}}$		
1	$\square_c, (\mathbf{1}, \mathbf{1})_0$	$(a, \bar{c}), (\mathbf{3}, \mathbf{1})_{-\frac{1}{3}}$	$(\bar{a}, \bar{c}), (\bar{\mathbf{3}}, \mathbf{1})_{\frac{1}{3}}$		
1	$\overline{\square}_c, (\mathbf{1}, \mathbf{1})_0$	$\overline{\square}_c, (\mathbf{1}, \mathbf{1})_0$	$(\bar{a}, c), (\bar{\mathbf{3}}, \mathbf{1})_{\frac{1}{3}}$	$(a, c), (\mathbf{3}, \mathbf{1})_{-\frac{1}{3}}$	
1	$\overline{\square}_c, (\mathbf{1}, \mathbf{1})_0$	$\overline{\square}_c, (\mathbf{1}, \mathbf{1})_0$	$(a, \bar{c}), (\mathbf{3}, \mathbf{1})_{-\frac{1}{3}}$	$(\bar{a}, \bar{c}), (\bar{\mathbf{3}}, \mathbf{1})_{\frac{1}{3}}$	
1	$(a, \bar{b}), (\mathbf{3}, \mathbf{2})_{\frac{1}{6}}$	$\square_a, (\bar{\mathbf{3}}, \mathbf{1})_{-\frac{2}{3}}$	$(b, c), (\mathbf{1}, \mathbf{2})_{-\frac{1}{2}}$	$(\bar{a}, \bar{c}), (\bar{\mathbf{3}}, \mathbf{1})_{\frac{1}{3}}$	$\square_b, (\mathbf{1}, \mathbf{1})_1$
1	$(\bar{b}, c), (\mathbf{1}, \mathbf{2})_{\frac{1}{2}}$	$(b, c), (\mathbf{1}, \mathbf{2})_{-\frac{1}{2}}$	$\overline{\square}_c, (\mathbf{1}, \mathbf{1})_0$	$(\bar{a}, c), (\bar{\mathbf{3}}, \mathbf{1})_{\frac{1}{3}}$	$(a, c), (\mathbf{3}, \mathbf{1})_{-\frac{1}{3}}$
1	$(\bar{b}, c), (\mathbf{1}, \mathbf{2})_{\frac{1}{2}}$	$(b, c), (\mathbf{1}, \mathbf{2})_{-\frac{1}{2}}$	$\overline{\square}_c, (\mathbf{1}, \mathbf{1})_0$	$(a, \bar{c}), (\mathbf{3}, \mathbf{1})_{-\frac{1}{3}}$	$(\bar{a}, \bar{c}), (\bar{\mathbf{3}}, \mathbf{1})_{\frac{1}{3}}$

Table 5. Matter additions and their multiplicity for the 41 three-node quivers with the non-Madrid embedding, up to five additions, and no vector pairs.

4 Statistics and Trends in Four-Node Quivers

There are a number of interesting physical issues that arise in the systematic study of four-node quivers with matter additions. We divide our analysis into two sections, with one studying the physics of the full set of quivers, and the other restricted to those quivers with a $U(1)'$ symmetry in the low energy theory. In each section, we will discuss them in the order of least to most phenomenological input.

Let us first summarize our assumptions. We study quivers with a number of different

hypercharge embeddings¹⁹, given by [17]

$$\begin{aligned}
U(1)_Y &= -\frac{1}{3}U(1)_a - \frac{1}{2}U(1)_b & U(1)_Y &= -\frac{1}{3}U(1)_a - \frac{1}{2}U(1)_b + \frac{1}{2}U(1)_d \\
U(1)_Y &= -\frac{1}{3}U(1)_a - \frac{1}{2}U(1)_b + U(1)_d & U(1)_Y &= \frac{1}{6}U(1)_a + \frac{1}{2}U(1)_c \\
U(1)_Y &= \frac{1}{6}U(1)_a + \frac{1}{2}U(1)_c + \frac{1}{2}U(1)_d & U(1)_Y &= \frac{1}{6}U(1)_a + \frac{1}{2}U(1)_c + \frac{3}{2}U(1)_d
\end{aligned} \tag{4.1}$$

where the generic $U(3)_a \times U(2)_b \times U(1)_c \times U(1)_d$ gauge symmetry is broken to the standard model (perhaps with additional $U(1)'$ factors) when anomalous $U(1)$ bosons receive a Stueckelberg mass. We construct all quivers with up to five additional matter fields beyond the exact MSSM and ensure that there are no vector pairs, where vector here means with respect to all symmetries in the theory, including the anomalous $U(1)$'s. We place no restrictions on the structure of matter additions except that they be in the class of fields which can be realized in the quiver (bifundamental, symmetric, and antisymmetric representations) and that the MSSM quiver plus additions satisfies the constraints (2.1), (2.2) and (2.3). The possible fields for each embedding are listed in tables 12-17 in appendix B.

4.1 Trends in Generic Quivers and Preferred Matter Additions

There are 146 three-node quivers and 89818 four-node quivers which satisfy our assumptions. Given the hypercharge embeddings that we study, these quivers have no phenomenological input other than that they contain the spectrum of the exact MSSM. Since the matter beyond the MSSM is entirely determined by the stringy constraints on chiral matter, it is interesting to see if any particular type of matter field is “preferred”. We present the results of this analysis in Table 6, where matter additions are classified with respect to their representation under the standard model gauge group.

MSSM singlets are by far the most preferred type of matter addition, with $SU(2)$ triplets with $Y = 0$ and quasichiral pairs (such as Higgs/lepton doublets and isosinglet down-type quarks) being the next most common, corroborating the intuition gained in the three-node case. We emphasize that all of these vector pairs are quasichiral with respect to the anomalous $U(1)$'s. This is because our analysis does not allow for the appearance of pairs which are vector with respect to all symmetries in the theory for the reasons stated above.

Some of the four-node quivers involve fields with the “wrong” relation between their $SU(2)$ and Y assignments, viz,

$$(\mathbf{1}, \mathbf{2})_0 \quad (\mathbf{1}, \mathbf{1})_{\frac{1}{2}} \quad (\mathbf{1}, \mathbf{1})_{-\frac{1}{2}} \quad (\mathbf{3}, \mathbf{1})_{\frac{1}{6}} \quad (\bar{\mathbf{3}}, \mathbf{1})_{-\frac{1}{6}}. \tag{4.2}$$

These have fractional electric charge, or, for the color triplets, would bind to form fractionally-charged color singlets. Such fractional charges are a typical feature of string constructions,

¹⁹This is the subset of hypercharge embeddings which can accommodate consistent quivers exhibiting the MSSM plus three singlets.

SM Rep	Total Multiplicity	Int. El.	4 th Gen. Removed	Shifted 4 th Gen. Also Removed
$(\mathbf{1}, \mathbf{1})_0$	174276	173578	173578	173578
$(\mathbf{1}, \mathbf{3})_0$	48291	48083	48083	48083
$(\mathbf{1}, \mathbf{2})_{-\frac{1}{2}}$	39600	39560	38814	38814
$(\mathbf{1}, \mathbf{2})_{\frac{1}{2}}$	38854	38814	38814	38814
$(\mathbf{\bar{3}}, \mathbf{1})_{\frac{1}{3}}$	25029	25007	24261	24241
$(\mathbf{3}, \mathbf{1})_{-\frac{1}{3}}$	24299	24277	24277	24241
$(\mathbf{1}, \mathbf{1})_1$	15232	15228	14482	14482
$(\mathbf{1}, \mathbf{1})_{-1}$	14486	14482	14482	14482
$(\mathbf{\bar{3}}, \mathbf{1})_{-\frac{2}{3}}$	3501	3501	2755	2755
$(\mathbf{3}, \mathbf{1})_{\frac{2}{3}}$	2755	2755	2755	2755
$(\mathbf{3}, \mathbf{2})_{\frac{1}{6}}$	1784	1784	1038	1038
$(\mathbf{\bar{3}}, \mathbf{2})_{-\frac{1}{6}}$	1038	1038	1038	1038
$(\mathbf{1}, \mathbf{2})_0$	852	0	0	0
$(\mathbf{1}, \mathbf{2})_{\frac{3}{2}}$	220	220	220	184
$(\mathbf{1}, \mathbf{2})_{-\frac{3}{2}}$	204	204	204	184
$(\mathbf{1}, \mathbf{1})_{\frac{1}{2}}$	152	0	0	0
$(\mathbf{1}, \mathbf{1})_{-\frac{1}{2}}$	152	0	0	0
$(\mathbf{3}, \mathbf{1})_{\frac{1}{6}}$	124	0	0	0
$(\mathbf{\bar{3}}, \mathbf{1})_{-\frac{1}{6}}$	124	0	0	0
$(\mathbf{3}, \mathbf{1})_{-\frac{4}{3}}$	36	36	36	0
$(\mathbf{1}, \mathbf{3})_{-1}$	36	36	36	0
$(\mathbf{\bar{3}}, \mathbf{2})_{\frac{5}{6}}$	36	36	36	0
$(\mathbf{\bar{3}}, \mathbf{1})_{\frac{4}{3}}$	20	20	20	0
$(\mathbf{1}, \mathbf{3})_1$	20	20	20	0
$(\mathbf{3}, \mathbf{2})_{-\frac{5}{6}}$	20	20	20	0

Table 6. Displayed are the standard model representation of matter additions, together with their total multiplicity across all three-node and four-node quivers in the hypercharge embeddings we have studied. The third column excludes quivers involving states that would lead to fractionally-charged color singlets. The fourth further excludes those where the matter additions correspond to a fourth generation, while the last also excludes a shifted fourth generation. The remaining additions correspond to MSSM singlets, $SU(2)$ triplets with $Y = 0$, and quasichiral pairs.

and are very strongly constrained by experiment unless they are extremely heavy or confined. (For a recent discussion, see, [64].) We therefore eliminate these quivers from further consideration.

Since we consider up to five matter additions beyond the MSSM, one possible set is a fourth generation $q_L, u_L^c, d_L^c, L, e_L^c$. This is not the only possible chiral anomaly-free set of five fields, though, as the combination

$$(\mathbf{3}, \mathbf{2})_{-\frac{5}{6}} \quad (\mathbf{\bar{3}}, \mathbf{1})_{\frac{1}{3}} \quad (\mathbf{\bar{3}}, \mathbf{1})_{\frac{4}{3}} \quad (\mathbf{1}, \mathbf{2})_{-\frac{3}{2}} \quad (\mathbf{1}, \mathbf{3})_1 \quad (4.3)$$

also turns out to be anomaly free. We call this combination (or its charge conjugate) a

“shifted fourth family”. It consists of a quark doublet with electric charges $-1/3$ and $-4/3$ and a lepton doublet with charges $-1, -2$. The right-handed leptons²⁰ occur in an $SU(2)$ triplet with charges $0, -1, -2$. As far as we know this curious shifted family has not been commented on²¹. Relatively few of the quivers involve a fourth family, e.g., only 595 of the quivers with the four-node Madrid embedding $U(1)_Y = \frac{1}{6}U(1)_a + \frac{1}{2}U(1)_c + \frac{1}{2}U(1)_d$. Furthermore, as described in the introduction, ordinary or shifted chiral families lead to Landau poles at low energy, and also require delicate cancellations for precision electroweak physics. For these reasons, we will instead focus on the remaining quivers after these are removed.

The last column in Table 6 lists the number of quivers with each type of matter addition, after removing those with fractional charges or an ordinary or shifted fourth family. There is a clear symmetry in the multiplicity of matter additions which indicates that standard model vector pairs are common. For example, there are 38814 occurrences of both $(\mathbf{1}, \mathbf{2})_{-\frac{1}{2}}$ and $(\mathbf{1}, \mathbf{2})_{\frac{1}{2}}$ that are not a part of a fourth family. This pair, or any other standard model vector pair, must be quasichiral with respect to the anomalous $U(1)$ ’s in the quiver given the assumptions of our analysis, i.e., no true vector pairs are allowed. The fact that a pair X and \overline{X} are quasichiral implies that the MSSM gauge invariant term $X\overline{X}$ has net anomalous $U(1)$ charge, forbidding it in the superpotential. This term could be generated by non-perturbative effects, such as D-instantons, in which case the mass of the pair cannot be determined without a detailed global construction. However, it is also possible that XX couples to some singlet S so that SXX is allowed (perturbatively). In that case the mass would be generated by the VEV of S , which (if nonzero) would likely be at the electroweak-TeV scale.

The most common quasichiral pair, as shown in Table 6, is a $(\mathbf{1}, \mathbf{2})_{-\frac{1}{2}} + (\mathbf{1}, \mathbf{2})_{\frac{1}{2}}$ pair, which could be Higgs-like or lepton-like, or both, depending on their allowed couplings. The next most common are down-type quark isosinglets, $(\overline{\mathbf{3}}, \mathbf{1})_{\frac{1}{3}} + (\mathbf{3}, \mathbf{1})_{-\frac{1}{3}}$, similar to those which occur in E_6 models [66, 67]. Together, these two types of pairs have the MSSM quantum numbers of $\mathbf{5} + \mathbf{5}^*$ of $SU(5)$. There are 2832 quivers where the additional matter has the quantum numbers of $\mathbf{5} + \mathbf{5}^*$ or $\mathbf{5} + \mathbf{5}^* + \mathbf{1}$. Also occurring, but less frequently, are isoscalar up-type pairs $(\overline{\mathbf{3}}, \mathbf{1})_{-\frac{2}{3}} + (\mathbf{3}, \mathbf{1})_{\frac{2}{3}}$, and isodoublet quark pairs, $(\overline{\mathbf{3}}, \mathbf{2})_{-\frac{1}{6}} + (\mathbf{3}, \mathbf{2})_{\frac{1}{6}}$. Such quasichiral quarks can be produced prolifically by QCD processes at the LHC, with a variety of interesting decay signatures [33–38]. Typically, the heavier quasichiral quarks and their scalar partners undergo cascade decays into the lighter ones, with the lightest decaying by mixing, leptoquark or diquark interactions, or higher-dimensional operators. The decays may be rapid, delayed, or stable (on collider time scales). Finally, there are a significant number of quivers with isosinglet charged lepton or Higgs pairs, $(\mathbf{1}, \mathbf{1})_1 + (\mathbf{1}, \mathbf{1})_{-1}$, and a few with isodoublets with electric charges $(\pm 1, \pm 2)$, $(\mathbf{1}, \mathbf{2})_{\frac{3}{2}} + (\mathbf{1}, \mathbf{2})_{-\frac{3}{2}}$.

In the MSSM the lepton doublets L and the down-type Higgs doublet H_d have the same gauge quantum numbers, transforming as $(\mathbf{1}, \mathbf{2})_{-\frac{1}{2}}$. If there is no distinction between

²⁰Similar to the ν_L^c for an ordinary family, a singlet $(1, 1)_0$ could be added phenomenologically to partner with the neutral component, but this would require the study of a sixth addition.

²¹A vector pair $(3, 2)_{-5/6} + (\overline{3}, 2)_{5/6}$ has been suggested in connection with the LEP forward-backward asymmetry into $b\overline{b}$ [65], but no such pairs emerge in our analysis.

them in an $\mathcal{N} = 1$ gauge theory, and therefore also in the quivers we study, then the lepton and R-parity violating couplings $q_L L d_L^c$, $L L e_L^c$, $H_u L$, and $S_\mu H_u L$ (where S_μ is a possible MSSM-singlet addition) are indistinguishable from the ordinary couplings $q_L H_d d_L^c$, $L H_d e_L^c$, μ -term $H_u H_d$, or dynamical μ -term $S_\mu H_u H_d$, and therefore are expected to be present at comparable scales. For example, if one is allowed perturbatively, then so is the other, although the coefficients could differ, with similar statements for non-perturbative terms generated by D-instantons. These do not necessarily lead to proton decay (depending on possible baryon number violation), but are nevertheless strongly constrained by limits on rare processes and other experiments [68, 69]. We therefore impose the further phenomenological restriction of only considering those quivers in which there is a clear distinction between the lepton and down-Higgs doublets. (This is a necessary but not sufficient condition for lepton number and R-parity conservation.) This can occur when they are quiver distinct, i.e., they are realized at the intersection of different stacks of D-branes, and therefore have different anomalous $U(1)$ charges. Specifically, the quivers that we study must have between 4 and 9 fields transforming as $(\mathbf{1}, \mathbf{2})_{-\frac{1}{2}}$. We call a given field of this type an “ H_d candidate” if it is quiver distinct from at least three other $(\mathbf{1}, \mathbf{2})_{-\frac{1}{2}}$ fields, which we identify as lepton doublets L .

	Multiplicity of Quivers					
Hypercharge	Total	Int. El.	H_d Candidate	No 4th Gen	$S_\mu H_u H_d$	$\nu_L^c H_u L$
$(-\frac{1}{3}, -\frac{1}{2}, 0)$	41	41	0	0	0	0
$(\frac{1}{6}, 0, \frac{1}{2})$	105	105	0	0	0	0
$(-\frac{1}{3}, -\frac{1}{2}, 0, 0)$	6974	6974	4954	4938	1824	2066
$(-\frac{1}{3}, -\frac{1}{2}, 0, \frac{1}{2})$	70	0	0	0	0	0
$(-\frac{1}{3}, -\frac{1}{2}, 0, 1)$	4176	4176	1842	1792	0	80
$(\frac{1}{6}, 0, \frac{1}{2}, 0)$	480	16	0	0	0	0
$(\frac{1}{6}, 0, \frac{1}{2}, \frac{1}{2})$	77853	77853	54119	53654	16754	15524
$(\frac{1}{6}, 0, \frac{1}{2}, \frac{3}{2})$	265	265	0	0	0	0

Table 7. Counts of quiver with certain properties. The first column displays the coefficients of the $U(1)$ factors for the hypercharge. The second gives the total number of quivers satisfying string constraints on chiral matter and anomaly cancellation, given the assumptions of our analysis. The third represents those quivers in the second column that do not give rise to fractional electric charge. The fourth represents those quivers in the third column that have a candidate for a distinct H_d . The remaining columns represent those quivers in column four where the matter additions do not correspond to a fourth family, those which have a singlet S_μ with a perturbative $S_\mu H_u H_d$ term, and those which have at least one ν_L^c -candidate with a Dirac coupling $\nu_L^c H_u L$ term, respectively.

We also consider possible couplings of the MSSM singlets, $(\mathbf{1}, \mathbf{1})_0$. A singlet S_μ is “NMSSM-like” if the anomalous $U(1)$ charges allow the perturbative coupling $S_\mu H_u H_d$ (independent of other discrete, global, or gauge symmetries that may affect S_μ). A dynamical μ term is generated if S_μ acquires a vacuum expectation value, as in the NMSSM and related models [12, 70–73]. Of course, an effective μ parameter can instead be generated by non-perturbative D-instantons [18, 19]. On the other hand, a singlet ν_L^c is “ ν_L^c -like”

if the coupling $\nu_L^c L H_u$ is perturbatively allowed. This can lead to the Dirac neutrino mass term that is needed in a conventional seesaw model, perhaps with the Majorana mass generated by D-instantons [18, 19]. (Other possibilities for small neutrino masses in this context include small non-perturbative Dirac masses [26], or a non-perturbative Weinberg operator [28], either being generated by D-instantons.) In a given quiver there may be multiple singlets, multiple up-type H_u fields/candidates, multiple H_d candidates, and/or more than four leptons L . The determination of whether a given singlet is NMSSM-like, ν_L^c -like, or neither must be done separately for each identification of the H_u , H_d , and L fields, and the possibilities iterated for each quiver. It is possible that a given quiver identification contains both NMSSM and ν_L^c -like singlets, but a given singlet cannot be both simultaneously if the H_d and L are distinct.

The quiver counts for each hypercharge embedding are presented in Table 7, where the various columns list the total number of quivers, those with fractional charges removed, and those which also have a unique H_d candidate. The last three columns impose the further respective restrictions of no fourth family, the existence of an NMSSM-type singlet, and the existence of at least one ν_L^c -type singlet (for at least one identification of the matter fields in the quiver). The four-node Madrid and non-Madrid embeddings, with $U(1)_Y = \frac{1}{6}U(1)_a + \frac{1}{2}U(1)_c + \frac{1}{2}U(1)_d$ and $U(1)_Y = -\frac{1}{3}U(1)_a - \frac{1}{2}U(1)_b$ respectively, give rise to almost ninety percent of the quivers, with the four-node Madrid embedding alone yielding over eighty percent. This is because they allow for the MSSM fields to be realized in more ways than the other embeddings. Only two of the hypercharge embeddings give rise to quivers with fractional electric charge.

Only three of the embeddings allow for a distinct H_d candidate²², because there must be at least two possible realizations of $(\mathbf{1}, \mathbf{2})_{-\frac{1}{2}}$ distinct from the vector partner of the H_u . For example, in a four-node quiver in the Madrid embedding with H_u transforming as (b, c) , H_d or L could be realized as (b, \bar{c}) , (b, \bar{d}) or (\bar{b}, \bar{d}) . More hypercharge embeddings could give rise to an H_d candidate in an analysis where vector pairs are allowed, but at the expense of introducing a perturbative μ -term, which is string scale at generic points in moduli space.

It is clear from Table 7 that most quivers do not have a fourth family. Combinatorically, this is simply because there are a very large number of ways to construct five-particle additions (though many do not give consistent quivers) and a relatively small number of those possibilities correspond to a fourth family.

Of the 60915 quivers with integral electric charge and a distinct H_d candidate, 51743 contain at least one MSSM singlet. In 25399 there is either a singlet S_μ which can couple as $S_\mu H_u H_d$, at least one singlet ν_L^c which can couple as $\nu_L^c H_u L$, or both. There are 2322 quivers that have both S_μ and ν_L^c -type singlets.

4.2 Quivers with $U(1)'$

We now restrict our attention to the subset of quivers in section 4.1 with a massless non-anomalous $U(1)'$ gauge symmetry in the low energy theory. This can occur if there is

²²It was already emphasized in section 3 that the three-node quivers do not distinguish H_d from L .

another linear combination,

$$U(1)' = \sum_x q_x U(1)_x, \quad (4.4)$$

which satisfies the constraints (2.2) and (2.3), and is therefore left massless by the generalized Green-Schwarz mechanism, just as in the case of the hypercharge boson. As stated in the introduction, these sometimes can emerge from non-abelian factors in larger $U(n)$ stacks (similar to the non-Madrid three-stack embedding of hypercharge), while in other cases they are not associated with underlying non-abelian symmetries (analogous to the Madrid three-stack hypercharge embedding). We again emphasize that these $U(1)'$ s may be present at the TeV scale and observable at the LHC, acquiring mass when singlet fields S (such as NMSSM-like singlets discussed previously) acquire a TeV scale VEV. The Z' does not obtain a string scale Stueckelberg mass and may therefore be observable even for large string scales M_s close to the Planck scale. This is in contrast to the scenario [47–54] in which anomalous bosons are only observable for M_s at the TeV scale. For recent globally consistent type IIA models with an extra non-anomalous $U(1)'$ symmetry, see [74], for example.

The number of quivers for each hypercharge embedding are listed in Table 8. Comparing with Table 7 we see that only a small fraction of the total involve a $U(1)'$, and most of those are in the four-node Madrid embedding. The table also lists the number with an H_d candidate, and the numbers with NMSSM-like or ν_L^c -like singlets²³. The three families of quarks and leptons are frequently quiver distinct. This can lead to family non-universal $U(1)'$ charges for two or more families of q_L , L , u_L^c , d_L^c , or e_L^c . In fact, it can be seen in Table 8 that only about 30% of the $U(1)'$ quivers are completely universal. Family non-universality leads to a violation of the GIM mechanism and therefore to flavor changing neutral current (FCNC) couplings of the Z' when fermion family mixing is turned on [76], and also to new constraints on the possible Yukawa matrices. Non-universality between the third family and the first two is most likely (because of strong experimental constraints from the neutral kaons and from μ interactions/decays), and has been suggested as an explanation of possible anomalies in the neutral B system [39–44], where even a heavy Z' exchange may be important because it is a tree-level exchange competing with SM or MSSM loop effects.

The multiplicities of matter additions in the $U(1)'$ models are listed in Table 9. One sees a pattern similar to the general case. There are large numbers of MSSM singlets and isotriplets with $Y = 0$, a few ordinary and shifted fourth families, and many quasichiral quark and lepton pairs, including some lepton doublets with charges $(\pm 1, \pm 2)$. In this case, the pairs may be chiral under either the non-anomalous $U(1)'$ (as is familiar in E_6 models), the anomalous ones, or both.

²³The possibilities for neutrino mass in $U(1)'$ models are surveyed in [75], but a detailed quiver analysis is beyond the scope of this paper.

Hypercharge	Multiplicity of Quivers			
	$U(1)'$	H_d Candidate	Fam. Univ	$S_\mu H_u H_d$ $LH_u \nu_L^c$
$(-\frac{1}{3}, -\frac{1}{2}, 0)$	0	0	0	0
$(\frac{1}{6}, 0, \frac{1}{2})$	1	0	0	0
$(-\frac{1}{3}, -\frac{1}{2}, 0, 0)$	198	146	56	70
$(-\frac{1}{3}, -\frac{1}{2}, 0, \frac{1}{2})$	0	0	0	0
$(-\frac{1}{3}, -\frac{1}{2}, 0, 1)$	78	16	10	0
$(\frac{1}{6}, 0, \frac{1}{2}, 0)$	0	0	0	0
$(\frac{1}{6}, 0, \frac{1}{2}, \frac{1}{2})$	1803	1466	629	610
$(\frac{1}{6}, 0, \frac{1}{2}, \frac{3}{2})$	82	0	0	0

Table 8. Counts of $U(1)'$ quivers with certain properties. In all counts, we have already filtered out any quivers with fractional electric charge. The second column gives counts of $U(1)'$ quivers, and the third gives counts of those with at least one H_d candidate. The remaining columns are the numbers which in addition are family universal with respect to $U(1)'$, have a perturbative $S_\mu H_u H_d$ term, or have at least one ν_L^c with an $LH_u \nu_L^c$ term, respectively.

SM Rep	Total Multiplicity	4 th Gen. Removed	Shifted 4 th Gen. Also Removed
$(\mathbf{1}, \mathbf{1})_0$	4556	4556	4556
$(\mathbf{1}, \mathbf{3})_0$	1290	1290	1290
$(\mathbf{1}, \mathbf{2})_{-\frac{1}{2}}$	631	619	619
$(\mathbf{1}, \mathbf{2})_{\frac{1}{2}}$	619	619	619
$(\mathbf{\bar{3}}, \mathbf{1})_{\frac{1}{3}}$	478	466	458
$(\mathbf{3}, \mathbf{1})_{-\frac{1}{3}}$	458	458	458
$(\mathbf{1}, \mathbf{1})_1$	262	250	250
$(\mathbf{1}, \mathbf{1})_{-1}$	250	250	250
$(\mathbf{1}, \mathbf{2})_{-\frac{3}{2}}$	101	101	93
$(\mathbf{1}, \mathbf{2})_{\frac{3}{2}}$	93	93	93
$(\mathbf{3}, \mathbf{2})_{\frac{1}{6}}$	46	34	34
$(\mathbf{\bar{3}}, \mathbf{2})_{-\frac{1}{6}}$	34	34	34
$(\mathbf{\bar{3}}, \mathbf{1})_{-\frac{2}{3}}$	30	18	18
$(\mathbf{3}, \mathbf{1})_{\frac{2}{3}}$	18	18	18
$(\mathbf{1}, \mathbf{3})_1$	8	8	0
$(\mathbf{3}, \mathbf{2})_{-\frac{5}{6}}$	8	8	0
$(\mathbf{\bar{3}}, \mathbf{1})_{\frac{4}{3}}$	8	8	0

Table 9. Displayed are the standard model representation of matter additions for $U(1)'$ quivers, together with their total multiplicity across all three-node and four-node quivers in the hypercharge embeddings we have studied. In the second column we have already filtered out quivers with fractional electric charge. The third excludes quivers where the matter additions correspond to a fourth generation. The fourth column additionally excludes quivers where the matter additions correspond to a shifted fourth generation, from which it can be seen that the majority of additions correspond to quasichiral pairs.

5 Conclusions

In this paper we studied constraints on chiral matter which arise in certain types of string compactifications, with a focus on the implications for matter beyond the MSSM, which might be seen at the LHC. These constraints include the standard constraints for cancellation of non-abelian anomalies, which are also present in ordinary field theories or in field theories augmented by string-motivated concepts such as “anomalous” $U(1)$ factors and Chern-Simons terms. However, some are genuinely stringy conditions associated with $U(2)$ and $U(1)$ tadpole cancellation (which ensures the cancellation of D-brane charge in the compact extra dimensions via Gauss’ law), while others correspond to conditions necessary for a massless hypercharge boson.

We studied the impact of these stringy conditions as found in type IIA intersecting brane and related constructions. The local constructions²⁴ we study are described by quivers, which are graphs in which the nodes represent the gauge factors (e.g., $U(N)$ groups from stacks of D6 branes), and the directed edges represent chiral matter, which live at the brane intersections. The allowed representations are bifundamentals, symmetric or antisymmetric products, and adjoints (which are not restricted to the intersections). The $U(1)$ factors from the trace generators are generically “anomalous”, with their bosons acquiring string scale masses by the Stueckelberg mechanism and the anomalies cancelled by Chern-Simons terms. The “anomalous” $U(1)$ ’s survive as global symmetries on the low energy theory, restricting the allowed superpotential couplings at the perturbative level. (These otherwise forbidden couplings can sometimes be restored by exponentially-suppressed non-perturbative D-instanton effects.) However, one or more linear combinations of the anomalous $U(1)$ ’s (such as weak hypercharge) may be non-anomalous with a massless boson (before turning on the Higgs mechanism). We have studied the three-node ($U(3) \times U(2) \times U(1)$) and four-node ($U(3) \times U(2) \times U(1) \times U(1)$) models with the conditions: (a) There is a realistic embedding of weak hypercharge, with the corresponding $U(1)_Y$ satisfying the necessary masslessness conditions. (b) The matter spectrum includes that of the MSSM (without right-handed neutrinos), with up to five additional matter fields. (c) There are no vector pairs of fields, because pairs that are vector under both the low energy gauge symmetries and the “anomalous” $U(1)$ ’s are typically expected to acquire string-scale masses. Moreover, vector pairs automatically satisfy the stringy conditions. (d) The quiver must satisfy the stringy tadpole conditions. The results are:

- There are no consistent three-node quivers and very few four-node ones that do not have at least one matter addition beyond the MSSM fields. This motivates, within our framework and assumptions, the possibility that there may be additional matter accessible at the LHC.
- We present three illustrative examples which demonstrate the power of the constraints. One is a quiver with the spectrum of the MSSM which violates a stringy constraint, despite being free of any (currently known) field theoretic pathology. In

²⁴The local constructions we study satisfy the stringy conditions, which are necessary but not sufficient for global consistency.

two other examples, we present both an MSSM singlet and a standard model vector pair which (when added to a consistent quiver) force the hypercharge boson to obtain a Stueckelberg mass, despite the fact that these additions are rather mundane in field theory.

- The violation of string consistency conditions by quivers with the exact MSSM spectrum often takes a suggestive form. This is particularly true in the case of three-node quivers, where an inconsistent quiver with the MSSM spectrum violates only one of the six string consistency conditions²⁵. Given the specific structure of the violation, some matter additions are much more likely than others to render the quiver consistent. In this sense, these matter fields are “preferred” by the string conditions.
- The most common matter additions are MSSM singlets, isotriplets with $Y = 0$, or quasichiral pairs, i.e., nonchiral under the MSSM, but chiral under additional anomalous or non-anomalous $U(1)$ factors. The latter include lepton/Higgs doublets, down-type isosinglet quarks, isodoublet or up-type isosinglet quarks, nonabelian singlets with charge ± 1 , and lepton/Higgs doublets with charges $(\pm 1, \pm 2)$. There are also smaller numbers of other additions that are excluded or strongly constrained by experimental or theoretical considerations, including chiral ordinary or charge-shifted fourth families, or fields which lead to fractionally-charged color singlet states.
- Many of the four-node quivers allow a distinction between the down-type Higgs doublet and the lepton doublets, due to their different anomalous $U(1)$ charges (i.e., they are quiver distinct). This is a necessary condition for lepton number and R-parity conservation.
- A small fraction of the four-node quivers (and one three-node one) allow an additional $U(1)'$ gauge symmetry in the low-energy theory. In the majority of the cases at least one type of fermion field has family non-universal couplings, due to their different embeddings at the quiver level. This implies flavor changing neutral currents when fermion mixing is turned on, as is possibly relevant to neutral B system.
- Many of the quivers have MSSM-singlets S_μ with perturbative couplings $S_\mu H_u H_d$ that can lead to an NMSSM-type dynamical μ term. (μ terms may instead be generated by D-instantons in other cases.)
- Many of the quivers have at least one MSSM-singlet ν_L^c that can have a perturbative Dirac neutrino mass term $L H_d \nu_L^c$, which is needed for a conventional neutrino seesaw. (Other possibilities for neutrino mass include a non-perturbative Weinberg operator or a small Dirac mass.)

This work may be viewed as a small step in the exploration of the subset of the string landscape that is consistent with the standard model or MSSM but may include additional TeV scale physics observable at the LHC. We have used string consistency conditions as a

²⁵There are six conditions in the case of three-node quivers and eight in the case of four-node quivers.

guidepost for new physics and note that our analyses allow for the likely scenario that the string scale M_s is comparable to the Planck scale.

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A Stringy Constraints on Chiral Matter

Here we briefly describe the origin of the constraints on chiral matter that are necessary for tadpole cancellation and a massless hypercharge boson. They include the field-theoretic constraints for the absence of non-abelian anomalies, though there are some (stringy) constraints that are not present in field theory.

Though the constraints necessary for tadpole cancellation and a massless hypercharge boson have often been discussed in the context of type IIA orientifold compactifications with intersecting D6-branes, they are also applicable in other regions of the landscape. It is straightforward to show that the same constraints hold in the T-dual type IIB formulation, and it has been shown [13–15] that they also apply in the context of the non-geometric rational conformal field theory phase of type IIA. One might expect, via duality, that the constraints apply in even broader patches of the landscape.

We will see that some of the stringy constraints are related to the ability to distinguish between $\mathbf{2}$ and $\overline{\mathbf{2}}$, due to charge under the (trace) $U(1)$ of $U(2)$. One might therefore expect similar constraints to apply in the heterotic string with holomorphic vector bundles with abelian factors in their structure group [77]. Such compactifications are S-dual to Type I compactifications with D9-branes and D6-branes, which in turn are mirror symmetric to the type IIA intersecting D6-brane models that we typically have in mind [16]. It would also be interesting to see if these constraints appear directly in M-theory. Let us briefly sketch their string theoretic origin in the type IIA corner of the landscape.

In type IIA string theory, gauge theories live on D6-branes wrapping three-cycles²⁶ π in the internal dimensions (a Calabi-Yau manifold). In the presence of O6-planes, every

²⁶D6-branes and O6-planes in the compactifications we consider fill spacetime and wrap three-cycles in the extra dimensions, which make up a Calabi-Yau manifold. We denote cycles wrapped by D6-branes with lower case latin subscripts, and denote the cycle wrapped by the O6-planes as π_{O6} . The topological intersection number of two three-cycles π_1 and π_2 is denoted by $\pi_1 \circ \pi_2$, which satisfies $\pi_1 \circ \pi_2 = -\pi_2 \circ \pi_1$.

Representation	Multiplicity
$\begin{array}{c} \square \\ \square \end{array}_a$	$\frac{1}{2} (\pi_a \circ \pi'_a + \pi_a \circ \pi_{O6})$
$\begin{array}{c} \square \\ \square \end{array}_a$	$\frac{1}{2} (\pi_a \circ \pi'_a - \pi_a \circ \pi_{O6})$
$(\square_a, \bar{\square}_b)$	$\pi_a \circ \pi_b$
(\square_a, \square_b)	$\pi_a \circ \pi'_b$

Table 10. Representations and multiplicities for chiral matter at the intersection of two D6-branes.

D-brane has an “image” brane under the orientifold involution. Chiral matter appears at the intersection of two D6-branes, where the representation and multiplicity are given in Table 10. D-branes are sources for Ramond-Ramond flux, which must be cancelled in the compact internal dimensions since (heuristically) the flux lines have nowhere to go. By using Gauss’ Law in the internal dimensions, one can ensure the cancellation of Ramond-Ramond charge. This gives the tadpole cancellation conditions

$$\sum_b N_b (\pi_b + \pi_{b'}) = 4 \pi_{O6}, \quad (\text{A.1})$$

which constrain the three-cycles wrapped by the D-branes. The integer N_b is the number of D6-branes on π_b , and the image brane associated to a brane on π_b wraps the cycle $\pi_{b'}$. These conditions must be satisfied in a consistent type IIA string compactification. In the absence of orientifolds, one can intersect a D-brane on a cycle π_a with (A.1) to obtain

$$0 = \pi_a \circ \sum_b N_b \pi_b = \sum_b N_b (\#(a, \bar{b}) - \#(\bar{a}, b)), \quad (\text{A.2})$$

which requires that $\#a = \#\bar{a}$. Typically the gauge symmetry on π_a is $U(N_a)$, so that the condition (A.2) for $N_a > 2$ are the conditions necessary for the absence of $SU(N_a)^3$ triangle anomalies. On the other hand, for $N_a = 2$ and $N_a = 1$, there are no such anomalies, yet these are still stringy constraints necessary for tadpole cancellation. In the presence of orientifold planes, the full condition is

$$\begin{aligned} N_a \geq 2 : \quad & \#a - \#\bar{a} + (N_a + 4) (\#\square\square_a - \#\square\square_a) + (N_a - 4) (\#\square_a - \#\bar{\square}_a) = 0 \\ N_a = 1 : \quad & \#a - \#\bar{a} + (N_a + 4) (\#\square\square_a - \#\square\square_a) = 0 \pmod{3}, \end{aligned} \quad (\text{A.3})$$

where the mod 3 condition for the $N_a = 1$ case comes from the fact that there is no antisymmetric representation of $U(1)$. This particular constraint is not affected by the structure of the quark sector, as every bifundamental quark or antiquark contributes ± 3 to the $N_a = 1$ constraint. It is these $N_a = 2$ and $N_a = 1$ conditions that MSSM quivers discussed in section 3.1 and 3.2 violate. For more details of this derivation, we refer the reader to [23].

The quivers studied in this paper generically have anomalous $U(1)$ factors, which could have abelian and/or mixed anomalies. These anomalies are cancelled [2] by the generalized

Green-Schwarz mechanism due to the presence of Chern-Simons terms. One such Chern-Simons term is of the form $\int B \wedge \text{Tr}(F)$, where B is a 2-form and F is a gauge field strength. In addition to playing a role in anomaly cancellation, this term gives a Stuckelberg mass to the $U(1)$ gauge boson. However, sometimes these terms are absent for a linear combination $U(1)_G = \sum_x q_x U(1)_x$. This occurs if

$$\sum_x q_x N_x (\pi_x - \pi_{x'}) = 0, \quad (\text{A.4})$$

in which case the $U(1)$ gauge boson is massless (except perhaps for a mass obtained via the ordinary Higgs mechanism). As with the condition on D6-brane three-cycles necessary for tadpole cancellation, this condition places constraints on chiral matter:

$$-q_a N_a (\#\square_a - \#\overline{\square}_a + \#\boxplus_a - \#\overline{\boxplus}_a) + \sum_{x \neq a} q_x N_x (\#(a, \overline{x}) - \#(a, x)) = 0 \quad (\text{A.5})$$

for $N_a \geq 2$, and

$$-q_a \frac{\#(a) - \#(\overline{a}) + 8(\#\square_a) - \#\overline{\square}_a)}{3} + \sum_{x \neq a} q_x N_x (\#(a, \overline{x}) - \#(a, x)) = 0 \quad (\text{A.6})$$

for $N_a = 1$. These constraints are necessary for $U(1)_G$ to be left massless, but are not sufficient. In MSSM quivers, one such linear combination must be identifiable as hypercharge, and any additional linear combination that satisfies the constraints is a $U(1)'$ symmetry.

Let us describe an interesting case regarding Chern-Simons couplings and a non-anomalous $U(1)$. In addition to terms of the form $\int B \wedge F$, cancellation of abelian and mixed gauge anomalies requires the presence of terms $\int \phi F \wedge F$, and cancellation of mixed gravitational anomalies requires the terms $\int \phi R \wedge R$. If a theory is such that there are no abelian or mixed anomalies for some anomalous $U(1)$, then there cannot be both ϕ -type and B -type terms, as they would give rise to an anomaly rather than cancelling one. However, in the scenario where a B -type term is present but ϕ -type terms are not, the $U(1)$ boson can receive a Stuckelberg mass, despite being anomaly free. This unusual feature, where a non-anomalous $U(1)$ nevertheless becomes massive, has been realized in explicit string compactifications [61].

Let us end with a brief comment about the possibility of global $SU(2)$ anomalies [60]. Global consistency conditions of string theory ensure the absence of such anomalies, which is closely related to the fact that D-branes are classified by K-theory [78]. Since we do not have a global string embedding, it is interesting to address whether or not one could add matter to an MSSM quiver in a way consistent with the stringy constraints while introducing a global $SU(2)$ anomaly in the process. In the three-node Madrid embedding, it is easy to see from (3.3) that one can only add an even number of doublets, and thus there is no global $SU(2)$ anomaly due to matter additions. Generalizing this result, it is easy to argue that T_b is even for any MSSM quiver of the type described in this paper, and therefore one will never introduce a global $SU(2)$ anomaly by adding matter to an MSSM quiver in a way consistent with stringy constraints.

Transformation	T_a	T_b	T_c	M_a	M_b	M_c
$\square_a \quad (\mathbf{6}, \mathbf{1})_{\frac{1}{3}}$	7	0	0	$-\frac{1}{2}$	0	0
$\overline{\square}_a \quad (\overline{\mathbf{6}}, \mathbf{1})_{-\frac{1}{3}}$	-7	0	0	$\frac{1}{2}$	0	0
$\square_b \quad (\overline{\mathbf{3}}, \mathbf{1})_{\frac{1}{3}}$	-1	0	0	$-\frac{1}{2}$	0	0
$\overline{\square}_b \quad (\mathbf{3}, \mathbf{1})_{-\frac{1}{3}}$	1	0	0	$\frac{1}{2}$	0	0
$\square_b \quad (\mathbf{1}, \mathbf{3})_0$	0	6	0	0	0	0
$\overline{\square}_b \quad (\mathbf{1}, \overline{\mathbf{3}})_0$	0	-6	0	0	0	0
$\square_b \quad (\mathbf{1}, \mathbf{1})_0$	0	-2	0	0	0	0
$\overline{\square}_b \quad (\mathbf{1}, \overline{\mathbf{1}})_0$	0	2	0	0	0	0
$(b, c) \quad (\mathbf{1}, \mathbf{2})_{\frac{1}{2}}$	0	-1	2	0	$-\frac{1}{2}$	$-\frac{1}{3}$
$(b, \overline{c}) \quad (\mathbf{1}, \overline{\mathbf{2}})_{-\frac{1}{2}}$	0	1	-2	0	$\frac{1}{2}$	$\frac{1}{3}$
$(\overline{b}, c) \quad (\overline{\mathbf{1}}, \mathbf{2})_{\frac{1}{2}}$	0	1	2	0	$-\frac{1}{2}$	$-\frac{1}{3}$
$(\overline{b}, \overline{c}) \quad (\overline{\mathbf{1}}, \overline{\mathbf{2}})_{-\frac{1}{2}}$	0	-1	-2	0	$\frac{1}{2}$	$\frac{1}{3}$
$\square_c \quad (\mathbf{1}, \mathbf{1})_1$	0	0	5	0	0	$-\frac{4}{3}$
$\overline{\square}_c \quad (\mathbf{1}, \overline{\mathbf{1}})_{-1}$	0	0	-5	0	0	$\frac{4}{3}$
$(a, \overline{b}) \quad (\mathbf{3}, \mathbf{2})_{\frac{1}{6}}$	2	-3	0	0	$-\frac{1}{2}$	0
$(\overline{a}, b) \quad (\overline{\mathbf{3}}, \overline{\mathbf{2}})_{-\frac{1}{6}}$	-2	3	0	0	$\frac{1}{2}$	0
$(a, b) \quad (\mathbf{3}, \mathbf{2})_{\frac{1}{6}}$	2	3	0	0	$-\frac{1}{2}$	0
$(\overline{a}, \overline{b}) \quad (\overline{\mathbf{3}}, \overline{\mathbf{2}})_{-\frac{1}{6}}$	-2	-3	0	0	$\frac{1}{2}$	0
$(a, \overline{c}) \quad (\mathbf{3}, \mathbf{1})_{-\frac{1}{3}}$	1	0	-3	$\frac{1}{2}$	0	0
$(\overline{a}, c) \quad (\overline{\mathbf{3}}, \mathbf{1})_{\frac{1}{3}}$	-1	0	3	$-\frac{1}{2}$	0	0
$(a, c) \quad (\mathbf{3}, \mathbf{1})_{\frac{2}{3}}$	1	0	3	$-\frac{1}{2}$	0	-1
$(\overline{a}, \overline{c}) \quad (\overline{\mathbf{3}}, \mathbf{1})_{-\frac{2}{3}}$	-1	0	-3	$\frac{1}{2}$	0	1

Madrid embedding additions.

Transformation	T_a	T_b	T_c	M_a	M_b	M_c
$\square_a \quad (\mathbf{6}, \mathbf{1})_{-\frac{2}{3}}$	7	0	0	1	0	0
$\overline{\square}_a \quad (\overline{\mathbf{6}}, \mathbf{1})_{\frac{2}{3}}$	-7	0	0	-1	0	0
$\square_a \quad (\overline{\mathbf{3}}, \mathbf{1})_{-\frac{2}{3}}$	-1	0	0	1	0	0
$\overline{\square}_a \quad (\mathbf{3}, \mathbf{1})_{\frac{2}{3}}$	1	0	0	-1	0	0
$\square_b \quad (\mathbf{1}, \mathbf{3})_{-1}$	0	6	0	0	1	0
$\overline{\square}_b \quad (\mathbf{1}, \overline{\mathbf{3}})_1$	0	-6	0	0	-1	0
$\square_b \quad (\mathbf{1}, \mathbf{1})_{-1}$	0	-2	0	0	1	0
$\overline{\square}_b \quad (\mathbf{1}, \overline{\mathbf{1}})_1$	0	2	0	0	-1	0
$(b, c) \quad (\mathbf{1}, \mathbf{2})_{\frac{1}{2}}$	0	-1	2	0	0	-1
$(b, \overline{c}) \quad (\mathbf{1}, \overline{\mathbf{2}})_{-\frac{1}{2}}$	0	1	-2	0	0	1
$(\overline{b}, c) \quad (\overline{\mathbf{1}}, \mathbf{2})_{\frac{1}{2}}$	0	1	2	0	0	1
$(\overline{b}, \overline{c}) \quad (\overline{\mathbf{1}}, \overline{\mathbf{2}})_{-\frac{1}{2}}$	0	-1	-2	0	0	-1
$\square_c \quad (\mathbf{1}, \mathbf{1})_0$	0	0	5	0	0	0
$\overline{\square}_c \quad (\mathbf{1}, \overline{\mathbf{1}})_0$	0	0	-5	0	0	0
$(a, \overline{b}) \quad (\mathbf{3}, \mathbf{2})_{\frac{1}{6}}$	2	-3	0	-1	1	0
$(\overline{a}, b) \quad (\overline{\mathbf{3}}, \overline{\mathbf{2}})_{-\frac{1}{6}}$	-2	3	0	1	-1	0
$(a, b) \quad (\mathbf{3}, \mathbf{2})_{-\frac{5}{6}}$	2	3	0	1	1	0
$(\overline{a}, \overline{b}) \quad (\overline{\mathbf{3}}, \overline{\mathbf{2}})_{\frac{5}{6}}$	-2	-3	0	-1	-1	0
$(a, \overline{c}) \quad (\mathbf{3}, \mathbf{1})_{-\frac{1}{3}}$	1	0	-3	0	0	1
$(\overline{a}, c) \quad (\overline{\mathbf{3}}, \mathbf{1})_{\frac{1}{3}}$	-1	0	3	0	0	-1
$(a, c) \quad (\mathbf{3}, \mathbf{1})_{-\frac{1}{3}}$	1	0	3	0	0	1
$(\overline{a}, \overline{c}) \quad (\overline{\mathbf{3}}, \mathbf{1})_{\frac{1}{3}}$	-1	0	-3	0	0	-1

Non-Madrid embedding additions.

Table 11. All possible fields which might arise in quivers with $U(3)$, $U(2)$, and $U(1)$ nodes, for each three-node hypercharge embedding. Listed for each field are its transformation behavior in the quiver, under the MSSM gauge group, its T-charge and its M-charge. None of these three-node candidate fields can lead to color singlet particles with fractional electric charge.

B Matter Addition Tables

In this appendix we list all fields, together with their standard model representation, that can be realized in a 3-node or 4-node quiver with the hypercharge embeddings studied in this paper. The possibilities for the three-node Madrid and non-Madrid embedding are presented in Table 11 and the four-node possibilities are listed in Tables 12-17. Each table also lists the T-charge and M-charge of each field.

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Transformation	T_a	T_b	T_c	T_d	M_a	M_b	M_c	M_d
$\square\square_a \quad (\mathbf{6}, \mathbf{1})_{-\frac{2}{3}}$	7	0	0	0	1	0	0	0
$\overline{\square\square}_a \quad (\overline{\mathbf{6}}, \mathbf{1})_{\frac{2}{3}}$	-7	0	0	0	-1	0	0	0
$\square_a \quad (\overline{\mathbf{3}}, \mathbf{1})_{-\frac{2}{3}}$	-1	0	0	0	1	0	0	0
$\overline{\square}_a \quad (\mathbf{3}, \mathbf{1})_{\frac{2}{3}}$	1	0	0	0	-1	0	0	0
$\square\square_b \quad (\mathbf{1}, \mathbf{3})_{-1}$	0	6	0	0	0	1	0	0
$\overline{\square\square}_b \quad (\mathbf{1}, \mathbf{3})_1$	0	-6	0	0	0	-1	0	0
$\square_b \quad (\mathbf{1}, \mathbf{1})_{-1}$	0	-2	0	0	0	1	0	0
$\overline{\square}_b \quad (\mathbf{1}, \mathbf{1})_1$	0	2	0	0	0	-1	0	0
$\square\square_d \quad (\mathbf{1}, \mathbf{1})_0$	0	0	0	5	0	0	0	0
$\overline{\square\square}_d \quad (\mathbf{1}, \mathbf{1})_0$	0	0	0	-5	0	0	0	0
$(c, d) \quad (\mathbf{1}, \mathbf{1})_0$	0	0	1	-1	0	0	0	0
$(\bar{c}, d) \quad (\mathbf{1}, \mathbf{1})_0$	0	0	-1	1	0	0	0	0
$(c, d) \quad (\mathbf{1}, \mathbf{1})_0$	0	0	1	1	0	0	0	0
$(\bar{c}, \bar{d}) \quad (\mathbf{1}, \mathbf{1})_0$	0	0	-1	-1	0	0	0	0
$(\bar{b}, c) \quad (\mathbf{1}, \mathbf{2})_{\frac{1}{2}}$	0	-1	2	0	0	0	-1	0
$(b, \bar{c}) \quad (\mathbf{1}, \mathbf{2})_{-\frac{1}{2}}$	0	1	-2	0	0	0	1	0
$(b, c) \quad (\mathbf{1}, \mathbf{2})_{-\frac{1}{2}}$	0	1	2	0	0	0	1	0
$(\bar{b}, \bar{c}) \quad (\mathbf{1}, \mathbf{2})_{\frac{1}{2}}$	0	-1	-2	0	0	0	-1	0
$\square\square_c \quad (\mathbf{1}, \mathbf{1})_0$	0	0	5	0	0	0	0	0
$\overline{\square\square}_c \quad (\mathbf{1}, \mathbf{1})_0$	0	0	-5	0	0	0	0	0
$(a, d) \quad (\mathbf{3}, \mathbf{1})_{-\frac{1}{3}}$	1	0	0	-3	0	0	0	1
$(\bar{a}, d) \quad (\overline{\mathbf{3}}, \mathbf{1})_{\frac{1}{3}}$	-1	0	0	3	0	0	0	-1
$(a, d) \quad (\mathbf{3}, \mathbf{1})_{-\frac{1}{3}}$	1	0	0	3	0	0	0	1
$(\bar{a}, \bar{d}) \quad (\overline{\mathbf{3}}, \mathbf{1})_{\frac{1}{3}}$	-1	0	0	-3	0	0	0	-1
$(a, b) \quad (\mathbf{3}, \mathbf{2})_{\frac{1}{6}}$	2	-3	0	0	-1	1	0	0
$(\bar{a}, b) \quad (\overline{\mathbf{3}}, \mathbf{2})_{-\frac{1}{6}}$	-2	3	0	0	1	-1	0	0
$(a, b) \quad (\mathbf{3}, \mathbf{2})_{-\frac{5}{6}}$	2	3	0	0	1	1	0	0
$(\bar{a}, \bar{b}) \quad (\overline{\mathbf{3}}, \mathbf{2})_{\frac{5}{6}}$	-2	-3	0	0	-1	-1	0	0
$(a, \bar{c}) \quad (\mathbf{3}, \mathbf{1})_{-\frac{1}{3}}$	1	0	-3	0	0	0	1	0
$(\bar{a}, c) \quad (\overline{\mathbf{3}}, \mathbf{1})_{\frac{1}{3}}$	-1	0	3	0	0	0	-1	0
$(a, c) \quad (\mathbf{3}, \mathbf{1})_{-\frac{1}{3}}$	1	0	3	0	0	0	1	0
$(\bar{a}, \bar{c}) \quad (\overline{\mathbf{3}}, \mathbf{1})_{\frac{1}{3}}$	-1	0	-3	0	0	0	-1	0
$(b, \bar{d}) \quad (\mathbf{1}, \mathbf{2})_{-\frac{1}{2}}$	0	1	0	-2	0	0	0	1
$(\bar{b}, d) \quad (\mathbf{1}, \mathbf{2})_{\frac{1}{2}}$	0	-1	0	2	0	0	0	-1
$(b, d) \quad (\mathbf{1}, \mathbf{2})_{-\frac{1}{2}}$	0	1	0	2	0	0	0	1
$(\bar{b}, \bar{d}) \quad (\mathbf{1}, \mathbf{2})_{\frac{1}{2}}$	0	-1	0	-2	0	0	0	-1

Table 12. Possible fields which might arise in the 4-node quiver with hypercharge embedding $U(1)_Y = -\frac{1}{3}U(1)_a - \frac{1}{2}U(1)_b$.

Transformation	T_a	T_b	T_c	T_d	M_a	M_b	M_c	M_d
$\square\square_a \quad (\mathbf{6}, \mathbf{1})_{-\frac{2}{3}}$	7	0	0	0	1	0	0	0
$\overline{\square\square}_a \quad (\overline{\mathbf{6}}, \mathbf{1})_{\frac{2}{3}}$	-7	0	0	0	-1	0	0	0
$\square_a \quad (\mathbf{\overline{3}}, \mathbf{1})_{-\frac{2}{3}}$	-1	0	0	0	1	0	0	0
$\overline{\square}_a \quad (\mathbf{3}, \mathbf{1})_{\frac{2}{3}}$	1	0	0	0	-1	0	0	0
$\square\square_b \quad (\mathbf{1}, \mathbf{3})_{-1}$	0	6	0	0	0	1	0	0
$\overline{\square\square}_b \quad (\mathbf{1}, \mathbf{\overline{3}})_1$	0	-6	0	0	0	-1	0	0
$\square_b \quad (\mathbf{1}, \mathbf{1})_{-1}$	0	-2	0	0	0	1	0	0
$\overline{\square}_b \quad (\mathbf{1}, \mathbf{1})_1$	0	2	0	0	0	-1	0	0
$\square\square_d \quad (\mathbf{1}, \mathbf{1})_1$	0	0	0	5	0	0	0	$-\frac{4}{3}$
$\overline{\square\square}_d \quad (\mathbf{1}, \mathbf{1})_{-1}$	0	0	0	-5	0	0	0	$\frac{4}{3}$
$(c, d) \quad (\mathbf{1}, \mathbf{1})_{-\frac{1}{2}}$	0	0	1	-1	0	0	$\frac{1}{2}$	$\frac{1}{6}$
$(\overline{c}, d) \quad (\mathbf{1}, \mathbf{1})_{\frac{1}{2}}$	0	0	-1	1	0	0	$-\frac{1}{2}$	$-\frac{1}{6}$
$(c, \overline{d}) \quad (\mathbf{1}, \mathbf{1})_{\frac{1}{2}}$	0	0	1	1	0	0	$-\frac{1}{2}$	$-\frac{1}{6}$
$(\overline{c}, \overline{d}) \quad (\mathbf{1}, \mathbf{1})_{-\frac{1}{2}}$	0	0	-1	-1	0	0	$\frac{1}{2}$	$\frac{1}{6}$
$(\overline{b}, c) \quad (\mathbf{1}, \mathbf{2})_{\frac{1}{2}}$	0	-1	2	0	0	0	-1	0
$(b, \overline{c}) \quad (\mathbf{1}, \mathbf{2})_{-\frac{1}{2}}$	0	1	-2	0	0	0	1	0
$(b, c) \quad (\mathbf{1}, \mathbf{2})_{-\frac{1}{2}}$	0	1	2	0	0	0	1	0
$(\overline{b}, \overline{c}) \quad (\mathbf{1}, \mathbf{2})_{\frac{1}{2}}$	0	-1	-2	0	0	0	-1	0
$\square\square_c \quad (\mathbf{1}, \mathbf{1})_0$	0	0	5	0	0	0	0	0
$\overline{\square\square}_c \quad (\mathbf{1}, \mathbf{1})_0$	0	0	-5	0	0	0	0	0
$(a, \overline{d}) \quad (\mathbf{3}, \mathbf{1})_{-\frac{5}{6}}$	1	0	0	-3	$\frac{1}{2}$	0	0	$\frac{3}{2}$
$(\overline{a}, d) \quad (\overline{\mathbf{3}}, \mathbf{1})_{\frac{5}{6}}$	-1	0	0	3	$-\frac{1}{2}$	0	0	$-\frac{3}{2}$
$(a, d) \quad (\mathbf{3}, \mathbf{1})_{\frac{1}{6}}$	1	0	0	3	$-\frac{1}{2}$	0	0	$\frac{1}{2}$
$(\overline{a}, \overline{d}) \quad (\overline{\mathbf{3}}, \mathbf{1})_{-\frac{1}{6}}$	-1	0	0	-3	$\frac{1}{2}$	0	0	$-\frac{1}{2}$
$(a, \overline{b}) \quad (\mathbf{3}, \mathbf{2})_{\frac{1}{6}}$	2	-3	0	0	-1	1	0	0
$(\overline{a}, b) \quad (\overline{\mathbf{3}}, \mathbf{2})_{-\frac{1}{6}}$	-2	3	0	0	1	-1	0	0
$(a, b) \quad (\mathbf{3}, \mathbf{2})_{-\frac{5}{6}}$	2	3	0	0	1	1	0	0
$(\overline{a}, \overline{b}) \quad (\overline{\mathbf{3}}, \mathbf{2})_{\frac{5}{6}}$	-2	-3	0	0	-1	-1	0	0
$(a, \overline{c}) \quad (\mathbf{3}, \mathbf{1})_{-\frac{1}{3}}$	1	0	-3	0	0	0	1	0
$(\overline{a}, c) \quad (\overline{\mathbf{3}}, \mathbf{1})_{\frac{1}{3}}$	-1	0	3	0	0	0	-1	0
$(a, c) \quad (\mathbf{3}, \mathbf{1})_{-\frac{1}{3}}$	1	0	3	0	0	0	1	0
$(\overline{a}, \overline{c}) \quad (\overline{\mathbf{3}}, \mathbf{1})_{\frac{1}{3}}$	-1	0	-3	0	0	0	-1	0
$(b, \overline{d}) \quad (\mathbf{1}, \mathbf{2})_{-1}$	0	1	0	-2	0	$\frac{1}{2}$	0	$\frac{4}{3}$
$(\overline{b}, d) \quad (\mathbf{1}, \mathbf{2})_1$	0	-1	0	2	0	$-\frac{1}{2}$	0	$-\frac{4}{3}$
$(b, d) \quad (\mathbf{1}, \mathbf{2})_0$	0	1	0	2	0	$-\frac{1}{2}$	0	$\frac{2}{3}$
$(\overline{b}, \overline{d}) \quad (\mathbf{1}, \mathbf{2})_0$	0	-1	0	-2	0	$\frac{1}{2}$	0	$-\frac{2}{3}$

Table 13. Possible fields which might arise in the 4-node quiver with hypercharge embedding $U(1)_Y = -\frac{1}{3}U(1)_a - \frac{1}{2}U(1)_b + \frac{1}{2}U(1)_d$. The $(\mathbf{1}, \mathbf{1})_{\frac{1}{2}}$, $(\mathbf{1}, \mathbf{2})_{1,0}$, and $(\mathbf{3}, \mathbf{1})_{\frac{1}{6}, -\frac{5}{6}}$ states and their conjugates would lead to fractional charge.

Transformation	T_a	T_b	T_c	T_d	M_a	M_b	M_c	M_d
$\square\square_a \quad (\mathbf{6}, \mathbf{1})_{-\frac{2}{3}}$	7	0	0	0	1	0	0	0
$\overline{\square\square}_a \quad (\overline{\mathbf{6}}, \mathbf{1})_{\frac{2}{3}}$	-7	0	0	0	-1	0	0	0
$\square_a \quad (\mathbf{\overline{3}}, \mathbf{1})_{-\frac{2}{3}}$	-1	0	0	0	1	0	0	0
$\overline{\square}_a \quad (\mathbf{3}, \mathbf{1})_{\frac{2}{3}}$	1	0	0	0	-1	0	0	0
$\square\square_b \quad (\mathbf{1}, \mathbf{3})_{-1}$	0	6	0	0	0	1	0	0
$\overline{\square\square}_b \quad (\mathbf{1}, \mathbf{\overline{3}})_1$	0	-6	0	0	0	-1	0	0
$\square_b \quad (\mathbf{1}, \mathbf{1})_{-1}$	0	-2	0	0	0	1	0	0
$\overline{\square}_b \quad (\mathbf{1}, \mathbf{1})_1$	0	2	0	0	0	-1	0	0
$\square\square_d \quad (\mathbf{1}, \mathbf{1})_1$	0	0	0	5	0	0	0	$-\frac{4}{3}$
$\overline{\square\square}_d \quad (\mathbf{1}, \mathbf{1})_{-1}$	0	0	0	-5	0	0	0	$\frac{4}{3}$
$(c, d) \quad (\mathbf{1}, \mathbf{1})_{-\frac{1}{2}}$	0	0	1	-1	0	0	$\frac{1}{2}$	$\frac{1}{6}$
$(\overline{c}, d) \quad (\mathbf{1}, \mathbf{1})_{\frac{1}{2}}$	0	0	-1	1	0	0	$-\frac{1}{2}$	$-\frac{1}{6}$
$(c, d) \quad (\mathbf{1}, \mathbf{1})_{\frac{1}{2}}$	0	0	1	1	0	0	$-\frac{1}{2}$	$-\frac{1}{6}$
$(\overline{c}, d) \quad (\mathbf{1}, \mathbf{1})_{-\frac{1}{2}}$	0	0	-1	-1	0	0	$\frac{1}{2}$	$\frac{1}{6}$
$(\overline{b}, c) \quad (\mathbf{1}, \mathbf{2})_{\frac{1}{2}}$	0	-1	2	0	0	0	-1	0
$(b, \overline{c}) \quad (\mathbf{1}, \mathbf{2})_{-\frac{1}{2}}$	0	1	-2	0	0	0	1	0
$(b, c) \quad (\mathbf{1}, \mathbf{2})_{-\frac{1}{2}}$	0	1	2	0	0	0	1	0
$(\overline{b}, \overline{c}) \quad (\mathbf{1}, \mathbf{2})_{\frac{1}{2}}$	0	-1	-2	0	0	0	-1	0
$\square\square_c \quad (\mathbf{1}, \mathbf{1})_0$	0	0	5	0	0	0	0	0
$\overline{\square\square}_c \quad (\mathbf{1}, \mathbf{1})_0$	0	0	-5	0	0	0	0	0
$(a, d) \quad (\mathbf{3}, \mathbf{1})_{-\frac{5}{6}}$	1	0	0	-3	$\frac{1}{2}$	0	0	$\frac{3}{2}$
$(\overline{a}, d) \quad (\overline{\mathbf{3}}, \mathbf{1})_{\frac{5}{6}}$	-1	0	0	3	$-\frac{1}{2}$	0	0	$-\frac{3}{2}$
$(a, d) \quad (\mathbf{3}, \mathbf{1})_{\frac{1}{6}}$	1	0	0	3	$-\frac{1}{2}$	0	0	$\frac{1}{2}$
$(\overline{a}, d) \quad (\overline{\mathbf{3}}, \mathbf{1})_{-\frac{1}{6}}$	-1	0	0	-3	$\frac{1}{2}$	0	0	$-\frac{1}{2}$
$(a, b) \quad (\mathbf{3}, \mathbf{2})_{\frac{1}{6}}$	2	-3	0	0	-1	1	0	0
$(\overline{a}, b) \quad (\overline{\mathbf{3}}, \mathbf{2})_{-\frac{1}{6}}$	-2	3	0	0	1	-1	0	0
$(a, b) \quad (\mathbf{3}, \mathbf{2})_{-\frac{5}{6}}$	2	3	0	0	1	1	0	0
$(\overline{a}, b) \quad (\overline{\mathbf{3}}, \mathbf{2})_{\frac{5}{6}}$	-2	-3	0	0	-1	-1	0	0
$(a, \overline{c}) \quad (\mathbf{3}, \mathbf{1})_{-\frac{1}{3}}$	1	0	-3	0	0	0	1	0
$(\overline{a}, c) \quad (\overline{\mathbf{3}}, \mathbf{1})_{\frac{1}{3}}$	-1	0	3	0	0	0	-1	0
$(a, c) \quad (\mathbf{3}, \mathbf{1})_{-\frac{1}{3}}$	1	0	3	0	0	0	1	0
$(\overline{a}, \overline{c}) \quad (\overline{\mathbf{3}}, \mathbf{1})_{\frac{1}{3}}$	-1	0	-3	0	0	0	-1	0
$(b, d) \quad (\mathbf{1}, \mathbf{2})_{-1}$	0	1	0	-2	0	$\frac{1}{2}$	0	$\frac{4}{3}$
$(\overline{b}, d) \quad (\mathbf{1}, \mathbf{\overline{2}})_1$	0	-1	0	2	0	$-\frac{1}{2}$	0	$-\frac{4}{3}$
$(b, d) \quad (\mathbf{1}, \mathbf{2})_0$	0	1	0	2	0	$-\frac{1}{2}$	0	$\frac{2}{3}$
$(\overline{b}, d) \quad (\mathbf{1}, \mathbf{\overline{2}})_0$	0	-1	0	-2	0	$\frac{1}{2}$	0	$-\frac{2}{3}$

Table 14. Possible fields which might arise in the 4-node quiver with hypercharge embedding $U(1)_Y = -\frac{1}{3}U(1)_a - \frac{1}{2}U(1)_b + U(1)_d$. The $(\mathbf{1}, \mathbf{1})_{\frac{1}{2}}$, $(\mathbf{1}, \mathbf{2})_{1,0}$, and $(\mathbf{3}, \mathbf{1})_{\frac{1}{6}, -\frac{5}{6}}$ states and their conjugates would lead to fractional charge.

Transformation	T_a	T_b	T_c	T_d	M_a	M_b	M_c	M_d
\square_a $(\mathbf{6}, \mathbf{1})_{\frac{1}{3}}$	7	0	0	0	$-\frac{1}{2}$	0	0	0
$\overline{\square}_a$ $(\overline{\mathbf{6}}, \mathbf{1})_{-\frac{1}{3}}$	-7	0	0	0	$\frac{1}{2}$	0	0	0
\square_a $(\mathbf{3}, \mathbf{1})_{\frac{1}{3}}$	-1	0	0	0	$-\frac{1}{2}$	0	0	0
$\overline{\square}_a$ $(\mathbf{3}, \mathbf{1})_{-\frac{1}{3}}$	1	0	0	0	$\frac{1}{2}$	0	0	0
\square_b $(\mathbf{1}, \mathbf{3})_0$	0	6	0	0	0	0	0	0
$\overline{\square}_b$ $(\mathbf{1}, \mathbf{3})_0$	0	-6	0	0	0	0	0	0
\square_b $(\mathbf{1}, \mathbf{1})_0$	0	-2	0	0	0	0	0	0
$\overline{\square}_b$ $(\mathbf{1}, \mathbf{1})_0$	0	2	0	0	0	0	0	0
\square_d $(\mathbf{1}, \mathbf{1})_0$	0	0	0	5	0	0	0	0
$\overline{\square}_d$ $(\mathbf{1}, \mathbf{1})_0$	0	0	0	-5	0	0	0	0
(c, d) $(\mathbf{1}, \mathbf{1})_{\frac{1}{2}}$	0	0	1	-1	0	0	$-\frac{1}{6}$	$-\frac{1}{2}$
(\overline{c}, d) $(\mathbf{1}, \mathbf{1})_{-\frac{1}{2}}$	0	0	-1	1	0	0	$\frac{1}{6}$	$\frac{1}{2}$
(c, d) $(\mathbf{1}, \mathbf{1})_{\frac{1}{2}}$	0	0	1	1	0	0	$-\frac{1}{6}$	$-\frac{1}{2}$
(\overline{c}, d) $(\mathbf{1}, \mathbf{1})_{-\frac{1}{2}}$	0	0	-1	-1	0	0	$\frac{1}{6}$	$\frac{1}{2}$
(b, c) $(\mathbf{1}, \mathbf{2})_{\frac{1}{2}}$	0	-1	2	0	0	$-\frac{1}{2}$	$-\frac{1}{3}$	0
(b, \overline{c}) $(\mathbf{1}, \mathbf{2})_{-\frac{1}{2}}$	0	1	-2	0	0	$\frac{1}{2}$	$\frac{1}{3}$	0
(b, c) $(\mathbf{1}, \mathbf{2})_{\frac{1}{2}}$	0	1	2	0	0	$-\frac{1}{2}$	$-\frac{1}{3}$	0
(b, \overline{c}) $(\mathbf{1}, \mathbf{2})_{-\frac{1}{2}}$	0	-1	-2	0	0	$\frac{1}{2}$	$\frac{1}{3}$	0
\square_c $(\mathbf{1}, \mathbf{1})_1$	0	0	5	0	0	0	$-\frac{4}{3}$	0
$\overline{\square}_c$ $(\mathbf{1}, \mathbf{1})_{-1}$	0	0	-5	0	0	0	$\frac{4}{3}$	0
(a, d) $(\mathbf{3}, \mathbf{1})_{\frac{1}{6}}$	1	0	0	-3	0	0	0	$-\frac{1}{2}$
(\overline{a}, d) $(\overline{\mathbf{3}}, \mathbf{1})_{-\frac{1}{6}}$	-1	0	0	3	0	0	0	$\frac{1}{2}$
(a, d) $(\mathbf{3}, \mathbf{1})_{\frac{1}{6}}$	1	0	0	3	0	0	0	$-\frac{1}{2}$
$(\overline{a}, \overline{d})$ $(\overline{\mathbf{3}}, \mathbf{1})_{-\frac{1}{6}}$	-1	0	0	-3	0	0	0	$\frac{1}{2}$
(a, b) $(\mathbf{3}, \mathbf{2})_{\frac{1}{6}}$	2	-3	0	0	0	$-\frac{1}{2}$	0	0
(\overline{a}, b) $(\overline{\mathbf{3}}, \mathbf{2})_{-\frac{1}{6}}$	-2	3	0	0	0	$\frac{1}{2}$	0	0
(a, b) $(\mathbf{3}, \mathbf{2})_{\frac{1}{6}}$	2	3	0	0	0	$-\frac{1}{2}$	0	0
$(\overline{a}, \overline{b})$ $(\overline{\mathbf{3}}, \mathbf{2})_{-\frac{1}{6}}$	-2	-3	0	0	0	$\frac{1}{2}$	0	0
(a, \overline{c}) $(\mathbf{3}, \mathbf{1})_{-\frac{1}{3}}$	1	0	-3	0	$\frac{1}{2}$	0	0	0
(\overline{a}, c) $(\overline{\mathbf{3}}, \mathbf{1})_{\frac{1}{3}}$	-1	0	3	0	$-\frac{1}{2}$	0	0	0
(a, c) $(\mathbf{3}, \mathbf{1})_{\frac{2}{3}}$	1	0	3	0	$-\frac{1}{2}$	0	-1	0
$(\overline{a}, \overline{c})$ $(\overline{\mathbf{3}}, \mathbf{1})_{-\frac{2}{3}}$	-1	0	-3	0	$\frac{1}{2}$	0	1	0
(b, d) $(\mathbf{1}, \mathbf{2})_0$	0	1	0	-2	0	0	0	0
(b, d) $(\mathbf{1}, \mathbf{2})_0$	0	-1	0	2	0	0	0	0
(b, d) $(\mathbf{1}, \mathbf{2})_0$	0	1	0	2	0	0	0	0
(b, d) $(\mathbf{1}, \mathbf{2})_0$	0	-1	0	-2	0	0	0	0

Table 15. Possible fields which might arise in the 4-node quiver with hypercharge embedding $U(1)_Y = \frac{1}{6}U(1)_a + \frac{1}{2}U(1)_c$. The $(\mathbf{1}, \mathbf{1})_{\frac{1}{2}}$, $(\mathbf{1}, \mathbf{2})_0$, and $(\mathbf{3}, \mathbf{1})_{\frac{1}{6}}$ states and their conjugates would lead to fractional charge.

Transformation	T_a	T_b	T_c	T_d	M_a	M_b	M_c	M_d
\square_a $(\mathbf{6}, \mathbf{1})_{\frac{1}{3}}$	7	0	0	0	$-\frac{1}{2}$	0	0	0
$\overline{\square}_a$ $(\overline{\mathbf{6}}, \mathbf{1})_{-\frac{1}{3}}$	-7	0	0	0	$\frac{1}{2}$	0	0	0
\square_a $(\mathbf{3}, \mathbf{1})_{\frac{1}{3}}$	-1	0	0	0	$-\frac{1}{2}$	0	0	0
$\overline{\square}_a$ $(\mathbf{3}, \mathbf{1})_{-\frac{1}{3}}$	1	0	0	0	$\frac{1}{2}$	0	0	0
\square_b $(\mathbf{1}, \mathbf{3})_0$	0	6	0	0	0	0	0	0
$\overline{\square}_b$ $(\mathbf{1}, \mathbf{3})_0$	0	-6	0	0	0	0	0	0
\square_b $(\mathbf{1}, \mathbf{1})_0$	0	-2	0	0	0	0	0	0
$\overline{\square}_b$ $(\mathbf{1}, \mathbf{1})_0$	0	2	0	0	0	0	0	0
\square_d $(\mathbf{1}, \mathbf{1})_1$	0	0	0	5	0	0	0	$-\frac{4}{3}$
$\overline{\square}_d$ $(\mathbf{1}, \mathbf{1})_{-1}$	0	0	0	-5	0	0	0	$\frac{4}{3}$
(c, d) $(\mathbf{1}, \mathbf{1})_0$	0	0	1	-1	0	0	$\frac{1}{3}$	$-\frac{1}{3}$
(\bar{c}, d) $(\mathbf{1}, \mathbf{1})_0$	0	0	-1	1	0	0	$-\frac{1}{3}$	$\frac{1}{3}$
(c, d) $(\mathbf{1}, \mathbf{1})_1$	0	0	1	1	0	0	$-\frac{2}{3}$	$-\frac{2}{3}$
(\bar{c}, d) $(\mathbf{1}, \mathbf{1})_{-1}$	0	0	-1	-1	0	0	$\frac{2}{3}$	$\frac{2}{3}$
(b, c) $(\mathbf{1}, \mathbf{2})_{\frac{1}{2}}$	0	-1	2	0	0	$-\frac{1}{2}$	$-\frac{1}{3}$	0
(b, \bar{c}) $(\mathbf{1}, \mathbf{2})_{-\frac{1}{2}}$	0	1	-2	0	0	$\frac{1}{2}$	$\frac{1}{3}$	0
(b, c) $(\mathbf{1}, \mathbf{2})_{\frac{1}{2}}$	0	1	2	0	0	$-\frac{1}{2}$	$-\frac{1}{3}$	0
(\bar{b}, \bar{c}) $(\mathbf{1}, \mathbf{2})_{-\frac{1}{2}}$	0	-1	-2	0	0	$\frac{1}{2}$	$\frac{1}{3}$	0
\square_c $(\mathbf{1}, \mathbf{1})_1$	0	0	5	0	0	0	$-\frac{4}{3}$	0
$\overline{\square}_c$ $(\mathbf{1}, \mathbf{1})_{-1}$	0	0	-5	0	0	0	$\frac{4}{3}$	0
(a, d) $(\mathbf{3}, \mathbf{1})_{-\frac{1}{3}}$	1	0	0	-3	$\frac{1}{2}$	0	0	0
(\bar{a}, d) $(\overline{\mathbf{3}}, \mathbf{1})_{\frac{1}{3}}$	-1	0	0	3	$-\frac{1}{2}$	0	0	0
(a, d) $(\mathbf{3}, \mathbf{1})_{\frac{2}{3}}$	1	0	0	3	$-\frac{1}{2}$	0	0	-1
(\bar{a}, \bar{d}) $(\overline{\mathbf{3}}, \mathbf{1})_{-\frac{2}{3}}$	-1	0	0	-3	$\frac{1}{2}$	0	0	1
(a, b) $(\mathbf{3}, \mathbf{2})_{\frac{1}{6}}$	2	-3	0	0	0	$-\frac{1}{2}$	0	0
(\bar{a}, b) $(\overline{\mathbf{3}}, \mathbf{2})_{-\frac{1}{6}}$	-2	3	0	0	0	$\frac{1}{2}$	0	0
(a, b) $(\mathbf{3}, \mathbf{2})_{\frac{1}{6}}$	2	3	0	0	0	$-\frac{1}{2}$	0	0
(\bar{a}, \bar{b}) $(\overline{\mathbf{3}}, \mathbf{2})_{-\frac{1}{6}}$	-2	-3	0	0	0	$\frac{1}{2}$	0	0
(a, \bar{c}) $(\mathbf{3}, \mathbf{1})_{-\frac{1}{3}}$	1	0	-3	0	$\frac{1}{2}$	0	0	0
(\bar{a}, c) $(\overline{\mathbf{3}}, \mathbf{1})_{\frac{1}{3}}$	-1	0	3	0	$-\frac{1}{2}$	0	0	0
(a, c) $(\mathbf{3}, \mathbf{1})_{\frac{2}{3}}$	1	0	3	0	$-\frac{1}{2}$	0	-1	0
(\bar{a}, \bar{c}) $(\overline{\mathbf{3}}, \mathbf{1})_{-\frac{2}{3}}$	-1	0	-3	0	$\frac{1}{2}$	0	1	0
(b, d) $(\mathbf{1}, \mathbf{2})_{-\frac{1}{2}}$	0	1	0	-2	0	$\frac{1}{2}$	0	$\frac{1}{3}$
(\bar{b}, d) $(\mathbf{1}, \mathbf{2})_{\frac{1}{2}}$	0	-1	0	2	0	$-\frac{1}{2}$	0	$-\frac{1}{3}$
(b, d) $(\mathbf{1}, \mathbf{2})_{\frac{1}{2}}$	0	1	0	2	0	$-\frac{1}{2}$	0	$-\frac{1}{3}$
(\bar{b}, \bar{d}) $(\mathbf{1}, \mathbf{2})_{-\frac{1}{2}}$	0	-1	0	-2	0	$\frac{1}{2}$	0	$\frac{1}{3}$

Table 16. Possible fields which might arise in the 4-node quiver with hypercharge embedding $U(1)_Y = \frac{1}{6}U(1)_a + \frac{1}{2}U(1)_c + \frac{1}{2}U(1)_d$.

Transformation	T_a	T_b	T_c	T_d	M_a	M_b	M_c	M_d
\square_a $(\mathbf{6}, \mathbf{1})_{\frac{1}{3}}$	7	0	0	0	$-\frac{1}{2}$	0	0	0
$\overline{\square}_a$ $(\overline{\mathbf{6}}, \mathbf{1})_{-\frac{1}{3}}$	-7	0	0	0	$\frac{1}{2}$	0	0	0
\square_a $(\mathbf{3}, \mathbf{1})_{\frac{1}{3}}$	-1	0	0	0	$-\frac{1}{2}$	0	0	0
$\overline{\square}_a$ $(\mathbf{3}, \mathbf{1})_{-\frac{1}{3}}$	1	0	0	0	$\frac{1}{2}$	0	0	0
\square_b $(\mathbf{1}, \mathbf{3})_0$	0	6	0	0	0	0	0	0
$\overline{\square}_b$ $(\mathbf{1}, \mathbf{3})_0$	0	-6	0	0	0	0	0	0
\square_b $(\mathbf{1}, \mathbf{1})_0$	0	-2	0	0	0	0	0	0
$\overline{\square}_b$ $(\mathbf{1}, \mathbf{1})_0$	0	2	0	0	0	0	0	0
\square_d $(\mathbf{1}, \mathbf{1})_3$	0	0	0	5	0	0	0	-4
$\overline{\square}_d$ $(\mathbf{1}, \mathbf{1})_{-3}$	0	0	0	-5	0	0	0	4
(c, d) $(\mathbf{1}, \mathbf{1})_{-1}$	0	0	1	-1	0	0	$\frac{4}{3}$	0
(\overline{c}, d) $(\mathbf{1}, \mathbf{1})_1$	0	0	-1	1	0	0	$-\frac{4}{3}$	0
(c, d) $(\mathbf{1}, \mathbf{1})_2$	0	0	1	1	0	0	$-\frac{5}{3}$	-1
(\overline{c}, d) $(\mathbf{1}, \mathbf{1})_{-2}$	0	0	-1	-1	0	0	$\frac{5}{3}$	1
(b, c) $(\mathbf{1}, \mathbf{2})_{\frac{1}{2}}$	0	-1	2	0	0	$-\frac{1}{2}$	$-\frac{1}{3}$	0
(b, \overline{c}) $(\mathbf{1}, \mathbf{2})_{-\frac{1}{2}}$	0	1	-2	0	0	$\frac{1}{2}$	$\frac{1}{3}$	0
(b, c) $(\mathbf{1}, \mathbf{2})_{\frac{1}{2}}$	0	1	2	0	0	$-\frac{1}{2}$	$-\frac{1}{3}$	0
$(\overline{b}, \overline{c})$ $(\mathbf{1}, \mathbf{2})_{-\frac{1}{2}}$	0	-1	-2	0	0	$\frac{1}{2}$	$\frac{1}{3}$	0
\square_c $(\mathbf{1}, \mathbf{1})_1$	0	0	5	0	0	0	$-\frac{4}{3}$	0
$\overline{\square}_c$ $(\mathbf{1}, \mathbf{1})_{-1}$	0	0	-5	0	0	0	$\frac{4}{3}$	0
(a, d) $(\mathbf{3}, \mathbf{1})_{-\frac{4}{3}}$	1	0	0	-3	$\frac{3}{2}$	0	0	1
(\overline{a}, d) $(\overline{\mathbf{3}}, \mathbf{1})_{\frac{4}{3}}$	-1	0	0	3	$-\frac{3}{2}$	0	0	-1
(a, d) $(\mathbf{3}, \mathbf{1})_{\frac{5}{3}}$	1	0	0	3	$-\frac{3}{2}$	0	0	-2
(\overline{a}, d) $(\overline{\mathbf{3}}, \mathbf{1})_{-\frac{5}{3}}$	-1	0	0	-3	$\frac{3}{2}$	0	0	2
(a, b) $(\mathbf{3}, \mathbf{2})_{\frac{1}{6}}$	2	-3	0	0	0	$-\frac{1}{2}$	0	0
(\overline{a}, b) $(\overline{\mathbf{3}}, \mathbf{2})_{-\frac{1}{6}}$	-2	3	0	0	0	$\frac{1}{2}$	0	0
(a, b) $(\mathbf{3}, \mathbf{2})_{\frac{1}{6}}$	2	3	0	0	0	$-\frac{1}{2}$	0	0
$(\overline{a}, \overline{b})$ $(\overline{\mathbf{3}}, \mathbf{2})_{-\frac{1}{6}}$	-2	-3	0	0	0	$\frac{1}{2}$	0	0
(a, \overline{c}) $(\mathbf{3}, \mathbf{1})_{-\frac{1}{3}}$	1	0	-3	0	$\frac{1}{2}$	0	0	0
(\overline{a}, c) $(\overline{\mathbf{3}}, \mathbf{1})_{\frac{1}{3}}$	-1	0	3	0	$-\frac{1}{2}$	0	0	0
(a, c) $(\mathbf{3}, \mathbf{1})_{\frac{2}{3}}$	1	0	3	0	$-\frac{1}{2}$	0	-1	0
$(\overline{a}, \overline{c})$ $(\overline{\mathbf{3}}, \mathbf{1})_{-\frac{2}{3}}$	-1	0	-3	0	$\frac{1}{2}$	0	1	0
(b, d) $(\mathbf{1}, \mathbf{2})_{-\frac{3}{2}}$	0	1	0	-2	0	$\frac{3}{2}$	0	1
(\overline{b}, d) $(\mathbf{1}, \mathbf{2})_{\frac{3}{2}}$	0	-1	0	2	0	$-\frac{3}{2}$	0	-1
(b, d) $(\mathbf{1}, \mathbf{2})_{\frac{3}{2}}$	0	1	0	2	0	$-\frac{3}{2}$	0	-1
$(\overline{b}, \overline{d})$ $(\mathbf{1}, \mathbf{2})_{-\frac{3}{2}}$	0	-1	0	-2	0	$\frac{3}{2}$	0	1

Table 17. Possible fields which might arise in the 4-node quiver with hypercharge embedding $U(1)_Y = \frac{1}{6}U(1)_a + \frac{1}{2}U(1)_c + \frac{3}{2}U(1)_d$.